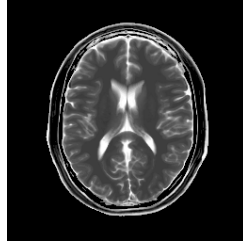


5 MRI (16 Points)

Recall that the measurements made by MRI machines are samples of the DFT $X[\cdot, \cdot]$ of some underlying image $x[\cdot, \cdot]$, which we recover via an inverse DFT. Recall also that, unlike many signals we have considered, this image's spatial domain representation is generally complex-valued.

Throughout this problem, we will consider the same 256×256 example image from lecture, for which $|x[\cdot, \cdot]|$ is shown below:



As we discussed in lecture and recitation, an important ongoing area of research involves attempting to faithfully reconstruct an image using as few samples of $X[\cdot, \cdot]$ as possible. In this problem, we will consider several different attempts to reduce the scan time of the image above.

5.1 Part 1

Consider reducing the scanning time by only sampling half of the rows of $X[\cdot, \cdot]$, creating a new image whose DFT is given by:

$$X_2[k_r, k_c] = \begin{cases} X[k_r, k_c] & \text{if } k_r \text{ is even} \\ 0 & \text{otherwise} \end{cases}$$

Which image on the facing page most closely matches $|x_2[\cdot, \cdot]|$, the magnitude of the spatial domain representation of this image?

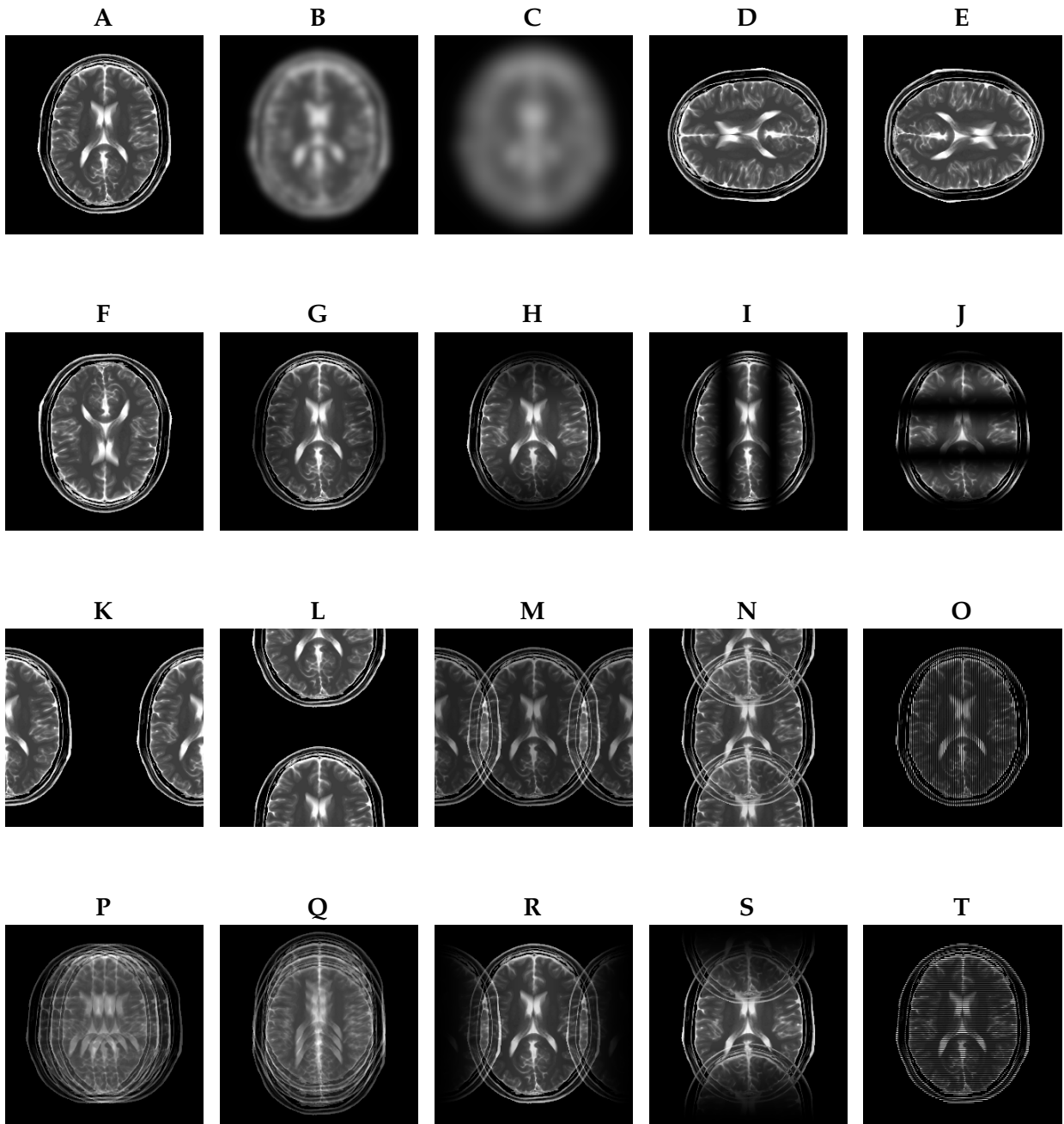
Enter a single letter:

Also consider a different approach, where we still only sample half of the rows, but we fill in the missing rows via linear interpolation rather than leaving them as 0's:

$$X_3[k_r, k_c] = \begin{cases} X[k_r, k_c] & \text{if } k_r \text{ is even} \\ X[k_r + 1, k_c]/2 + X[k_r - 1, k_c]/2 & \text{otherwise} \end{cases}$$

Which image on the facing page most closely matches $|x_3[\cdot, \cdot]|$, the magnitude of the spatial domain representation of this image?

Enter a single letter:



5.2 Part 2

Ben Bitdiddle suggests that a better way to cut down on scanning time would be to sample only half of the rows, but, particularly, to sample only where $0 \leq k_r \leq 128$, and then to use the conjugate symmetry of the DFT to fill in the missing values, i.e., for $-127 \leq k_r < 0$, set $X[k_r, k_c] = X^*[-k_r, -k_c]$.

Ben asserts that this approach will allow him to reconstruct $x[\cdot, \cdot]$ exactly, while only explicitly sampling half of the DFT coefficients.

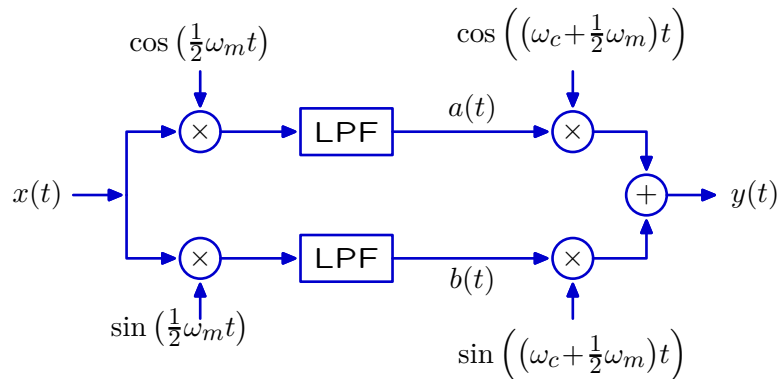
Is Ben's assertion true? **Yes** or **No**:

Briefly explain your reasoning:

Worksheet (intentionally blank)

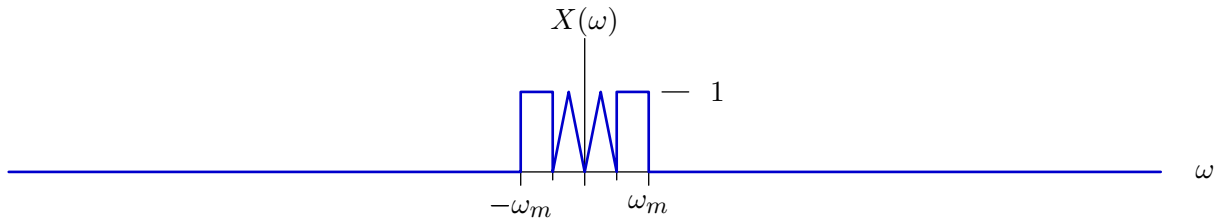
6 Modulation (14 Points)

Consider the following modulation scheme, where $\omega_c \gg \omega_m$.



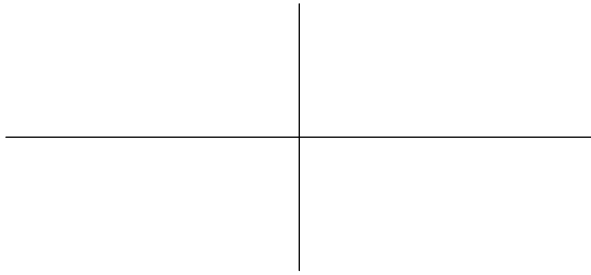
Assume that each lowpass filter (LPF) is ideal, with cutoff frequency $\omega_m/2$, and note that $a(\cdot)$ and $b(\cdot)$ represent the respective outputs of the two low-pass filters.

Also assume that the input signal has the following (purely real) Fourier transform:

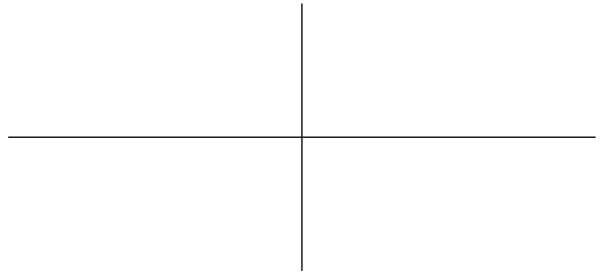


On the facing page, sketch the real and imaginary parts of $A(\omega)$, $B(\omega)$, and $Y(\omega)$. Label all important magnitudes and frequencies.

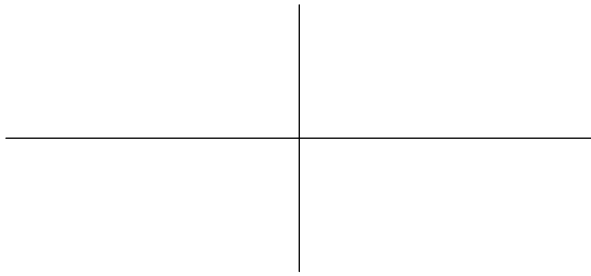
$\text{Re}(A(\omega))$



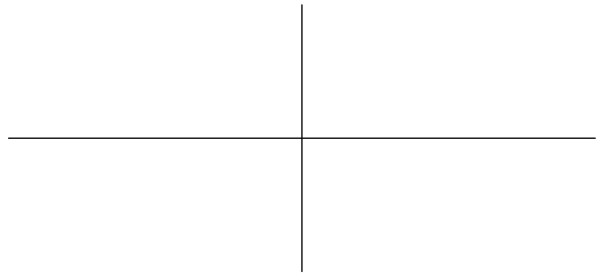
$\text{Im}(A(\omega))$



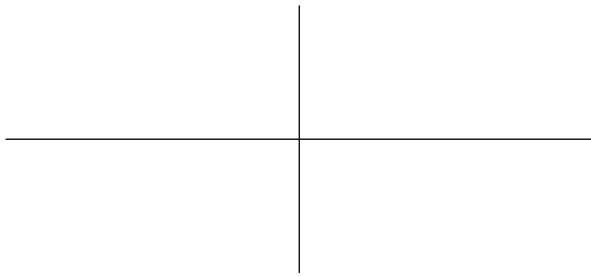
$\text{Re}(B(\omega))$



$\text{Im}(B(\omega))$



$\text{Re}(Y(\omega))$



$\text{Im}(Y(\omega))$

