

# 6.003: Signal Processing

## 2D Convolution

*November 16, 2021*

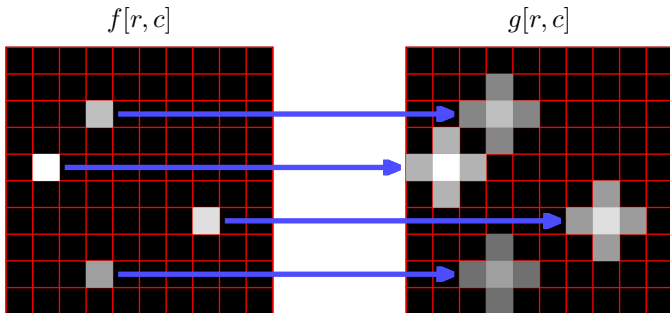
## 2D Convolution

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We can represent a system that is linear and shift-invariant by its unit-sample response (its response to a unit-sample signal):

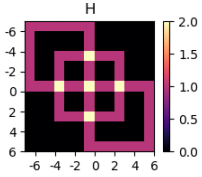
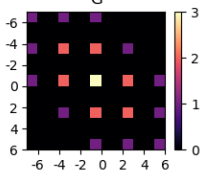
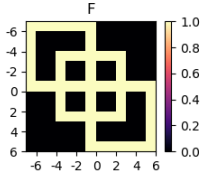
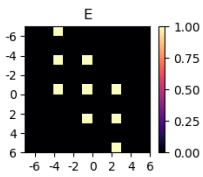
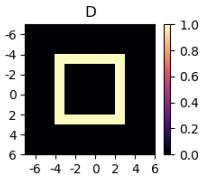
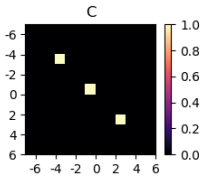
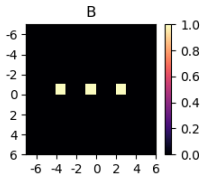
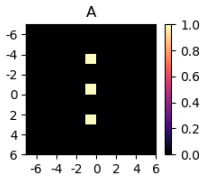
$$\delta[r, c] \rightarrow h[r, c]$$

The response of such a system to an input  $f[r, c]$  is the superposition of shifted and scaled versions of the unit-sample response.



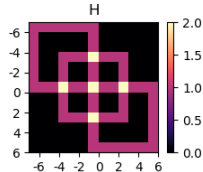
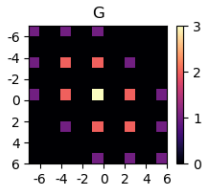
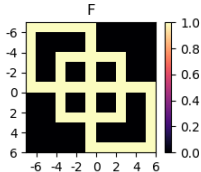
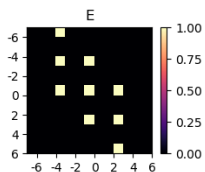
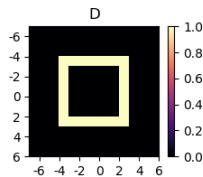
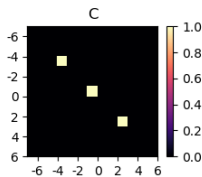
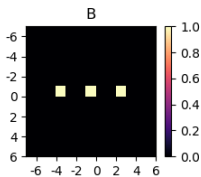
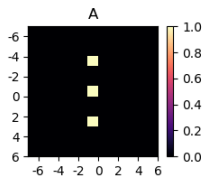
# 2D Convolution

Which of the following images can be constructed by convolving two of the other images?



## 2D Convolution

Which of the following images can be constructed by convolving two of the other images?



$$E = A * C$$

$$G = B * E$$

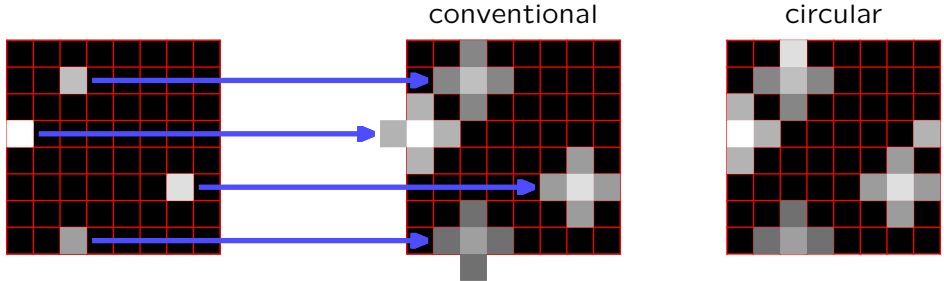
$$H = C * D$$

## 2D Circular Convolution

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Convolution in space is equivalent to multiplication of DTFT's.

However, multiplication of DFT's (or DTFS's) is equivalent to **circular convolution** in space.

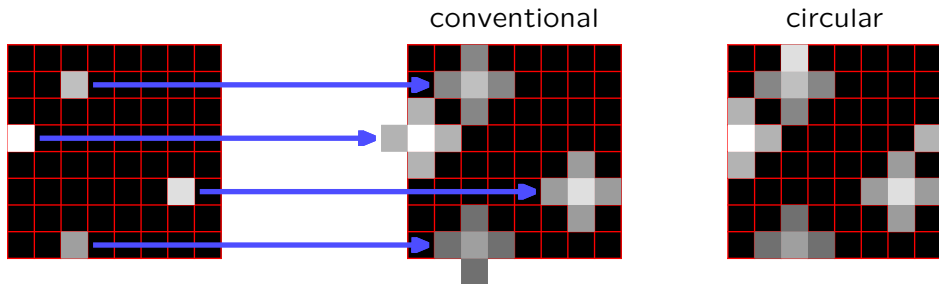


The domains of the input and output signals are limited by the dimensions of the DFTs.

## 2D Circular Convolution

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Two perspectives.

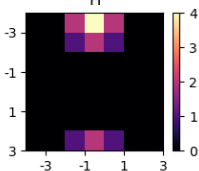
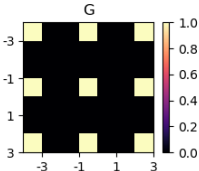
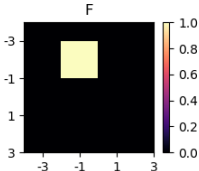
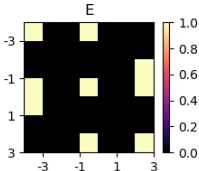
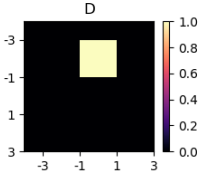
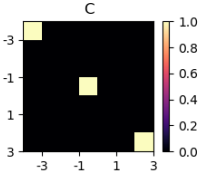
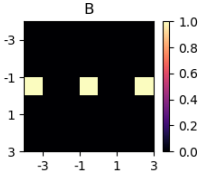
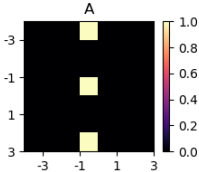


**Focusing on the output:** If part of the output image falls outside the region, move it back into the region by shifting that part by an integer number of widths or heights.

**Focusing on the input:** Start by periodically extending the input by repeating the region of interest to tile the entire plane. Then do conventional convolution.

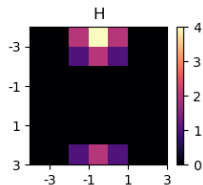
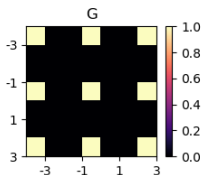
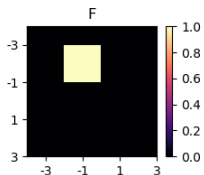
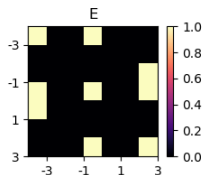
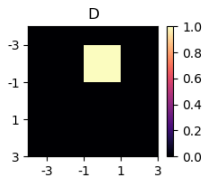
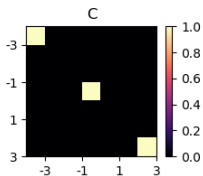
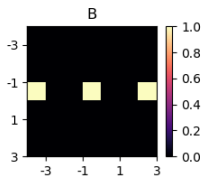
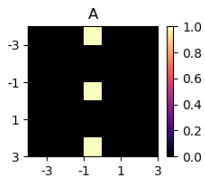
# 2D Circular Convolution

Which of the following images can be constructed by circularly convolving two of the other images?



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$$E = A \circledast C$$

$$G = A \circledast B$$

$$H = D \circledast F$$



## Picture Convolutions: Match an Expression to Each Image

$$g_1[r, c] = f[r, c] * \delta[r-20, c+50]$$

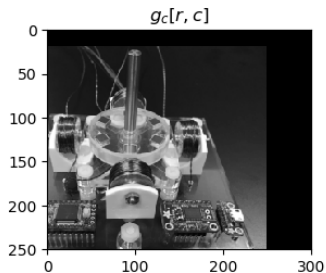
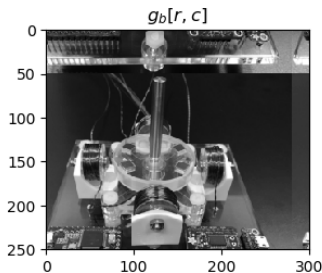
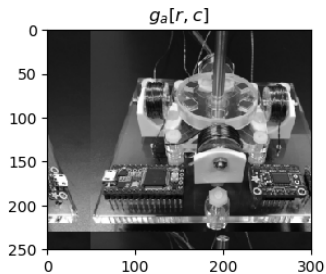
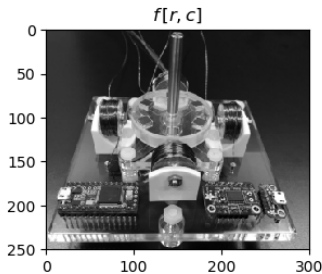
$$g_2[r, c] = f[r, c] * \delta[r+20, c-50]$$

$$g_3[r, c] = f[r, c] * \delta[r-50, c+20]$$

$$g_4[r, c] = f[r, c] \otimes \delta[r-20, c+50]$$

$$g_5[r, c] = f[r, c] \otimes \delta[r+20, c-50]$$

$$g_6[r, c] = f[r, c] \otimes \delta[r-50, c+20]$$



## Picture Convolutions: Match an Expression to Each Image

$$g_c[r, c] = f[r, c] * \delta[r-20, c+50]$$

$$g_2[r, c] = f[r, c] * \delta[r+20, c-50]$$

$$g_3[r, c] = f[r, c] * \delta[r-50, c+20]$$

$$g_4[r, c] = f[r, c] \otimes \delta[r-20, c+50]$$

$$g_a[r, c] = f[r, c] \otimes \delta[r+20, c-50]$$

$$g_b[r, c] = f[r, c] \otimes \delta[r-50, c+20]$$

