# 6.003: Signal Processing

## Two-Dimensional DFT

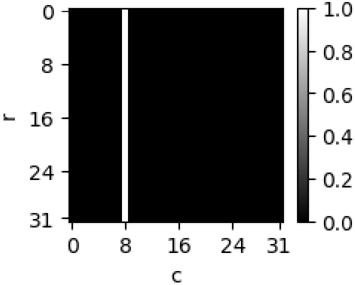
$$F[k_x,k_y]=\frac{1}{N_xN_y}\sum_{n_x=0}^{N_x-1}\sum_{n_y=0}^{N_y-1}f[n_x,n_y]\,e^{-j\left(\frac{2\pi k_x}{N_x}n_x+\frac{2\pi k_y}{N_y}n_y\right)}$$

$$f[n_x,n_y]=\sum_{k_x=0}^{N_x-1}\sum_{k_y=0}^{N_y-1}F[k_x,k_y]\,e^{j\left(\frac{2\pi k_x}{N_x}n_x+\frac{2\pi k_y}{N_y}n_y\right)}$$

*November 9, 2021*

# Simple Shapes

Find the 2D DFT of the following vertical bar.

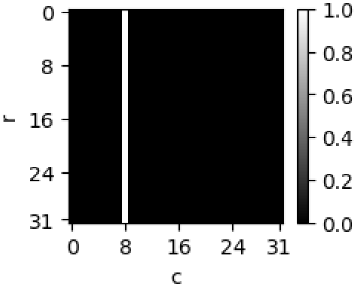



Array indices in numpy are `[r,c]`, where `r` is row and `c` is column.

The image is $32 \times 32$ pixels. The bar is at $c = 8$.

## Simple Shapes

Find the 2D DFT of the following vertical bar.



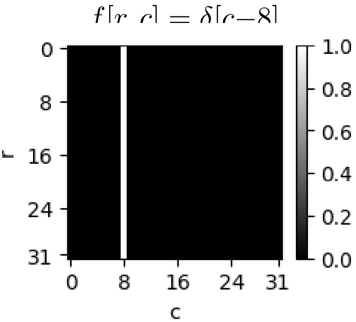

$$F[k_r,k_c] = \frac{1}{RC}\sum_{r=0}^{R-1}\sum_{c=0}^{C-1} f[r,c]\, e^{-j\left(\frac{2\pi k_r}{R}r+\frac{2\pi k_c}{C}c\right)}$$

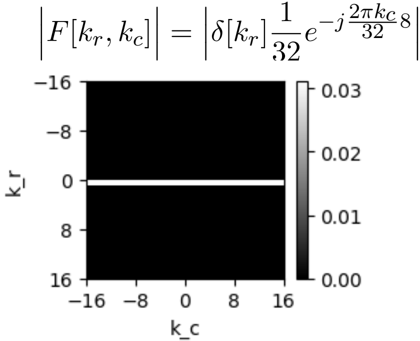
$$= \frac{1}{32^2}\sum_{r=0}^{31}\sum_{c=0}^{31} \delta[c-8]\, e^{-j\left(\frac{2\pi k_r}{32}r+\frac{2\pi k_c}{32}c\right)}$$

$$= \frac{1}{32}\sum_{r=0}^{31} e^{-j\frac{2\pi k_r}{32}r}\frac{1}{32}\sum_{c=0}^{31}\delta[c-8]e^{-j\frac{2\pi k_c}{32}c} = \delta[k_r]\frac{1}{32}e^{-j\frac{2\pi k_c}{32}8}$$

# Simple Shapes

Find the 2D DFT of the following vertical bar.




Frequency $[k_r, k_c]$ is often plotted with the origin in the center.

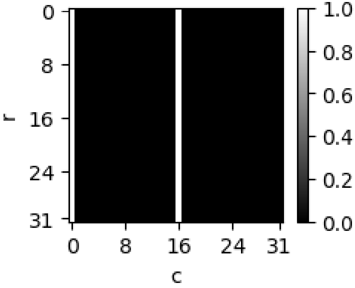
How does the $e^{-j\frac{2\pi k_c}{32}8}$ term contribute to the right panel?

Could you change $f[r,c]$ so that $F[k_r,k_c] = \frac{1}{32}\delta[k_r]$? (no exponential)

Could you change $f[r,c]$ so that the horizontal bar in $F$ is at $k_r = 8$?

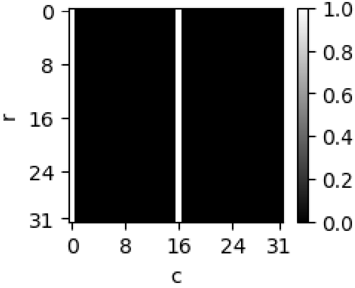
## Simple Shapes

Find the 2D DFT of this image, where bars are at $c=0$ and $c=16$.



# Simple Shapes

Find the 2D DFT of this image, where bars are at $c=0$ and $c=16$.



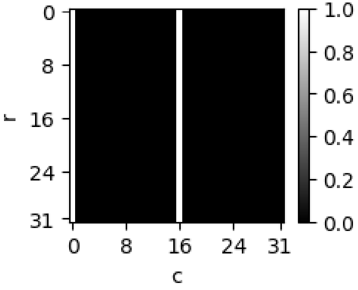

$$\delta[c] \quad \overset{\text{DFT}}{\Longleftrightarrow} \quad \frac{1}{32}\delta[k_r]$$

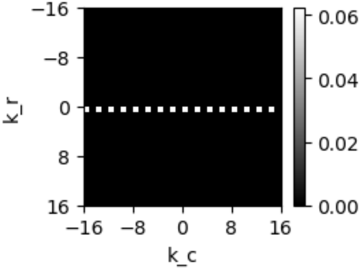
$$\delta[c]+\delta[c-16] \quad \overset{\text{DFT}}{\Longleftrightarrow} \quad \frac{1}{32}\delta[k_r]+\frac{1}{32}e^{-j\frac{2\pi k_c}{32}16}\delta[k_r]=\frac{1}{32}\left(1+(-1)^{k_c}\right)\delta[k_r]$$

$$=\begin{cases}\frac{1}{16} & \text{if } k_c \text{ is even and } k_r=0\\ 0 & \text{otherwise}\end{cases}$$

# Simple Shapes

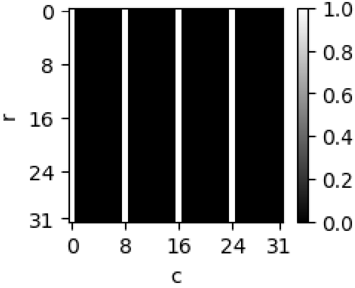
Find the 2D DFT of this image, where bars are at $c=0$ and $c=16$.



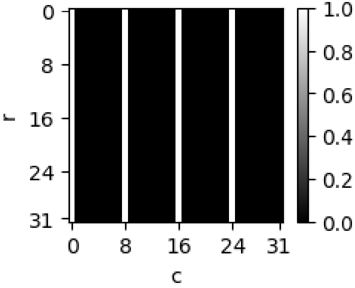
# Simple Shapes

Find the 2D DFT of the following image.



## Simple Shapes

Find the 2D DFT of the following image.




$$\delta[c] \overset{\text{DFT}}{\Longleftrightarrow} \frac{1}{32}\delta[k_r]$$

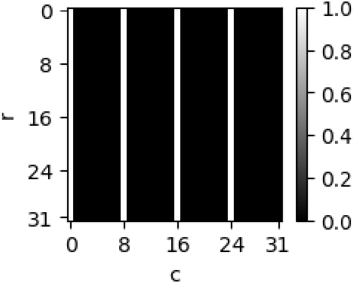
$$\sum_{m=0}^{3}\delta[c-8m] \overset{\text{DFT}}{\Longleftrightarrow} \frac{1}{32}\delta[k_r]\sum_{m=0}^{3} e^{-j\frac{2\pi k_c}{C}8m}$$

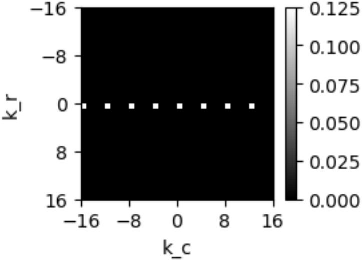
$$= \frac{1}{32}\delta[k_r]\sum_{m=0}^{3} e^{-j\frac{2\pi k_c}{4}m} = \frac{1}{8}\delta[k_r]\delta[k_c \bmod 4]$$

# Simple Shapes

Find the 2D DFT of the following image.

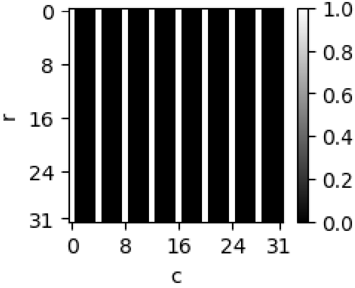
$$\sum_{m=0}^{3} \delta[c-8m] \quad \overset{\text{DFT}}{\Longleftrightarrow} \quad \frac{1}{8}\delta[k_r]\delta[k_c \bmod 4]$$



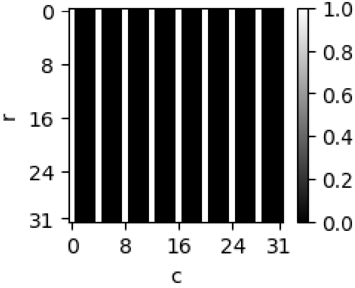
# Simple Shapes

Find the 2D DFT of the following image.



## Simple Shapes

Find the 2D DFT of the following image.



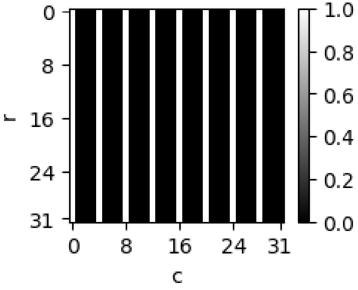

$$\delta[c] \quad \stackrel{\text{DFT}}{\Longleftrightarrow} \quad \frac{1}{32}\delta[k_r]$$

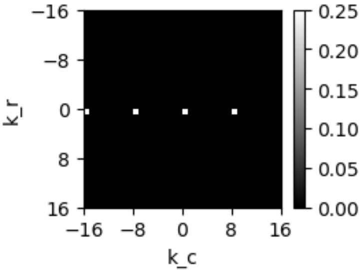
$$\sum_{m=0}^{7} \delta[c-4m] \quad \stackrel{\text{DFT}}{\Longleftrightarrow} \quad \frac{1}{32}\delta[k_r]\sum_{m=0}^{7} e^{-j\frac{2\pi k_c}{C}4m}$$

$$= \frac{1}{32}\delta[k_r]\sum_{m=0}^{7} e^{-j\frac{2\pi k_c}{8}m} = \frac{1}{4}\delta[k_r]\delta[k_c \bmod 8]$$

# Simple Shapes

Find the 2D DFT of the following image.




What's the relation between the period in space (left) and the period in frequency (right)?