6.003: Signal Processing

Quiz Review Questions

October 28, 2021

Continuous-Time Fourier Transform

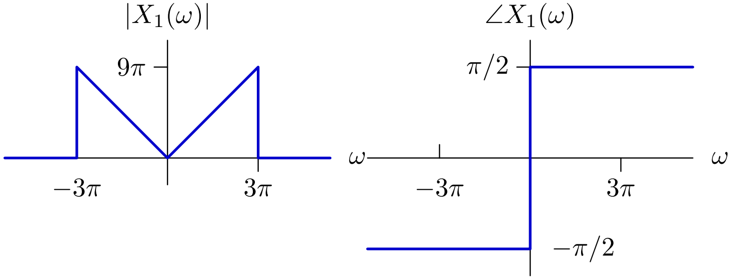
Property	$y(t)$	$Y(\omega)$
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(\omega) + bX_2(\omega)$
Time reversal	$x(-t)$	$X(-\omega)$
Time delay	$x(t - t_0)$	$e^{-j\omega t_0} X(\omega)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Scaling time	$x(at)$	$\frac{1}{\lvert a \rvert} X\left(\frac{\omega}{a}\right)$
Time derivative	$\frac{dx(t)}{dt}$	$j\omega X(\omega)$
Frequency derivative	$tx(t)$	$j \frac{d}{d\omega} X(\omega)$

Discrete-Time Fourier Transform

Property	$y[n]$	$Y(\Omega)$
Linearity	$ax_1[n] + bx_2[n]$	$aX_1(\Omega) + bX_2(\Omega)$
Time reversal	$x[-n]$	$X(-\Omega)$
Time delay	$x[n - n_0]$	$e^{-j\Omega n_0} X(\Omega)$
Conjugation	$x^*[n]$	$X^*(-\Omega)$
Frequency derivative	$nx[n]$	$j\frac{d}{d\Omega}X(\Omega)$

Fourier Transforms: Part 1

Determine the signal $x_1(t)$ whose Fourier transform $X_1(\omega)$ has the following magnitude and angle.



Express $x_1(t)$ as a closed-form function of time.

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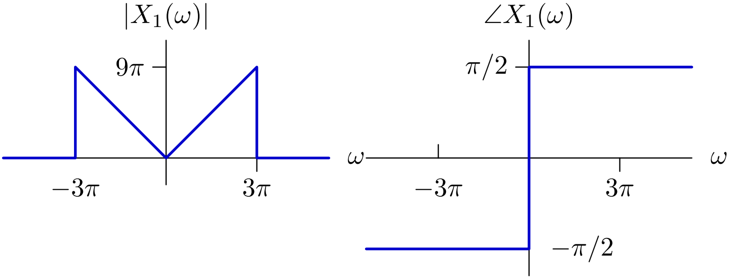


Express $x_1(t)$ as a closed-form function of time.

$$X_1(\omega) = \begin{cases} 3j\omega & |\omega| < 3\pi \\ 0 & \text{otherwise} \end{cases}$$

Fourier Transforms: Part 1 Continued

$$X_1(\omega) = \begin{cases} 3j\omega & |\omega| < 3\pi \\ 0 & \text{otherwise} \end{cases}$$

Let $X_{1a}(\omega)$ represent a rectangular pulse in frequency so that

$$X_{1a}(\omega) = \begin{cases} 1 & |\omega| < 3\pi \\ 0 & \text{otherwise} \end{cases}$$

Then $X_1(\omega) = 3j\omega X_{1a}(\omega)$ and therefore

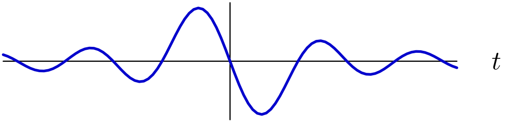
$$x_1(t) = 3\frac{d}{dt}x_{1a}(t)$$

where

$$x_{1a}(t) = \frac{\sin 3\pi t}{\pi t}$$

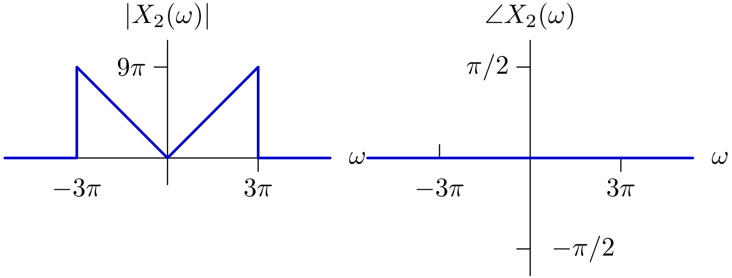
Therefore

$$x_1(t) = \frac{3}{\pi t^2}\left(3\pi t \cos 3\pi t - \sin 3\pi t\right)$$



Fourier Transforms: Part 2

Determine $x_2(t)$, whose Fourier transform $X_2(\omega)$ has the following magnitude and angle.



Express $x_2(t)$ as a closed-form function of time.

Fourier Transforms: Part 2 Continued

Consider just the right-hand side of $X_2(\omega)$:

$$X_r(\omega) = \begin{cases} 3\omega & \text{if } 0 < \omega < 3\pi \\ 0 & \text{otherwise} \end{cases}$$

$$x_r(t) = \frac{1}{2\pi} \int_0^{3\pi} 3\omega e^{j\omega t} d\omega$$

We could integrate by parts, but it would be easier to think about $x_r(t)$ as the time derivative of some other function $y(t)$. Taking the time derivative introduces a factor of $j\omega$ in the Fourier domain, so we would like to express $x_r(t)$ in a form that has a factor of $j\omega$:

$$x_r(t) = \frac{3}{j2\pi} \int_0^{3\pi} j\omega e^{j\omega t} d\omega$$

$$y(t) = \frac{3}{j2\pi} \int_0^{3\pi} e^{j\omega t} d\omega = \frac{3}{j2\pi t} \left(1 - e^{j3\pi t}\right)$$

Fourier Transforms: Part 2 Continued

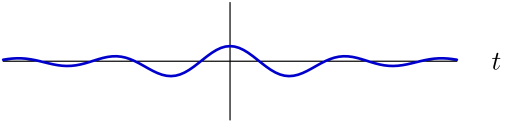
Using the product rule for derivatives, we have:

$$x_r(t) = \dot{y}(t) = \frac{3}{2\pi t}\left(-j3\pi\right)e^{j3\pi t} - \frac{3}{2\pi t^2}\left(1 - e^{j3\pi t}\right)$$

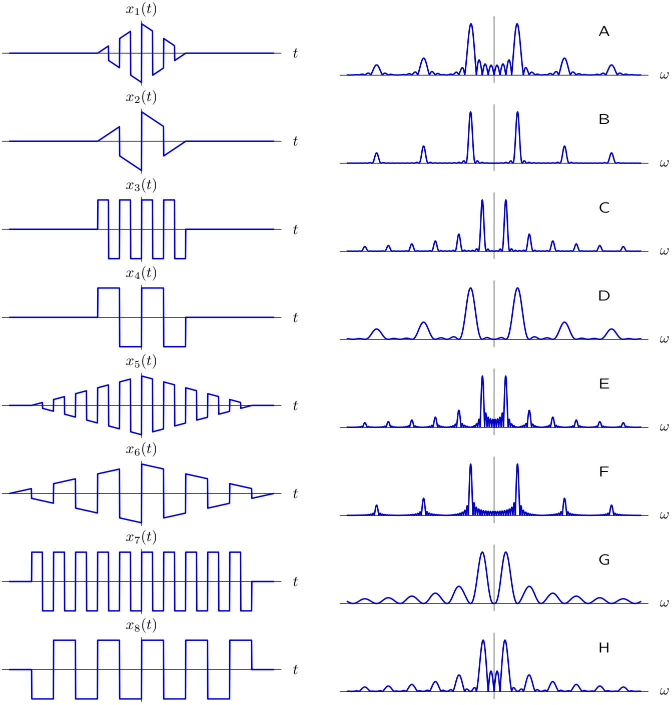
$$\begin{aligned} x(t) &= x_r(t) + x_r(-t) \\ &= -j\frac{9}{2t}e^{j3\pi t} + j\frac{9}{2t}e^{-j3\pi t} - \frac{3}{2\pi t^2}\left(1 - e^{j3\pi t}\right) - \frac{3}{2\pi t^2}\left(1 - e^{-j3\pi t}\right) \end{aligned}$$

Thus

$$x(t) = \frac{9}{t}\sin\left(3\pi t\right) - \frac{3}{\pi t^2} + \frac{3}{\pi t^2}\cos\left(3\pi t\right)$$



Match the Time Waveforms (left) with their CTFTs (right).



Match the Time Waveforms (left) with their CTFTs (right).

solutions: DGAHBCFE

The time waveforms can be classified as having three important parameters:

- period: those of x_1, x_3, x_5, and x_7 are half as long as those of others.
- shape: triangular or square and can be thought of as multiplying an underlying periodic time signal.
- overall length: x_1, x_2, x_3, and x_4 are short, the others are long.

Match the Time Waveforms (left) with their CTFTs (right).

Effects of parameters on magnitudes of FTs:

- The period in time is inversely related to the period in frequency. Thus A, B, D, and F (which have longer periods in frequency) correspond to x_1, x_3, x_5, and x_7.
- Since the shape multiplies the time waveform, it convolves with the frequency waveform. The shape is long compared to the periodicity of the time waveform, therefore the convolution affects the short timescale (i.e., between the periods). The square (in time) has more high frequencies than the triangle, so the square in frequency has larger overshoot. Thus A, E, F, and H correspond to squares (x_3, x_4, x_7, and x_8), and the others correspond to triangles.
- The overall length is long compared to the periodicity of the time waveform, therefore the convolution affects the short timescale (i.e., between the periods). The longer the shape, the shorter the spread around each lobe in the frequency domain. Therefore, the broad lobes (A, D, G, and H) correspond to the short overall lengths (x_1, x_2, x_3, and x_4).