

6.003: Signal Processing

Convolution and Filtering

time domain

$$y(t) = (h * x)(t) = \int h(\tau)x(t - \tau) d\tau$$

$$y[n] = (h * x)[n] = \sum_m h[m]x[n - m]$$

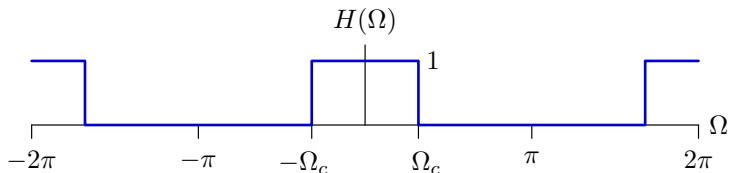
frequency domain

$$Y(\omega) = H(\omega)X(\omega)$$

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The “Ideal” Low-Pass Filter

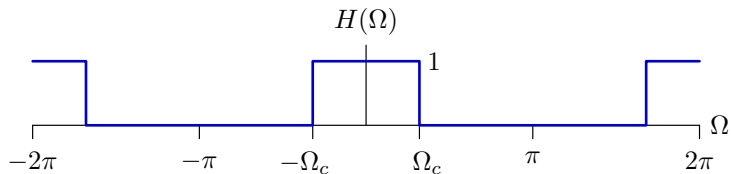
Consider a system characterized by the following purely real frequency response:



Such a system is called a **low-pass filter**, because it allows low frequencies to pass through unmodified, while attenuating high frequencies.

We could apply this filter to a signal by multiplying the DTFT of that signal by the values above. But we could also apply the filter by operating in the time domain.

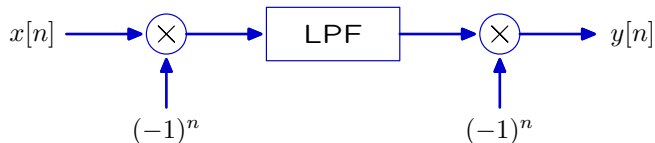
The “Ideal” Low-Pass Filter



We can apply this filter to a signal by convolving with its unit sample response. What is the unit sample response of the system whose frequency response is shown above?

Cascaded System

Consider the following system, where LPF represents a lowpass filter of the form discussed on the previous slides.



How many of the following statements are true?

- The transformation from $x[n]$ to $y[n]$ is linear.
- The transformation from $x[n]$ to $y[n]$ is time invariant.
- The transformation from $x[n]$ to $y[n]$ is a high-pass filter.