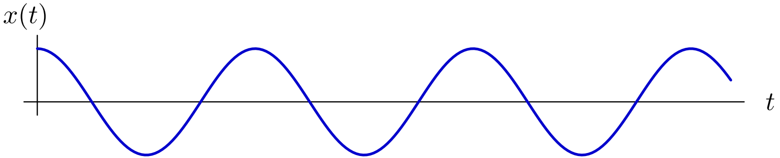
6.003: Signal Processing

Sampling and Aliasing

September 21, 2021

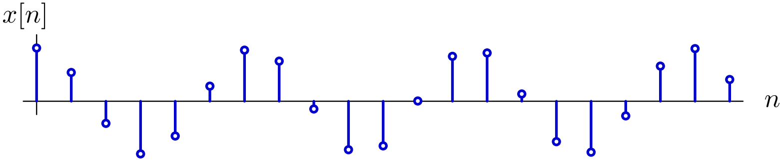
Tones and Sinusoids

A "tone" is a pressure that changes sinusoidally with time.



In 6.003, we will think of this as a "continuous-time" (CT) signal.

In contrast, a "discrete-time" (DT) signal is a sequence of numbers.



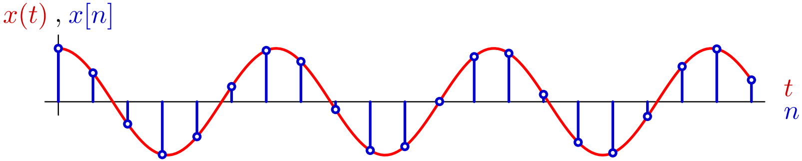
Mathematically:

$$x(t) = A\cos(\omega t)$$

$$x[n] = A\cos(\Omega n)$$

CT and DT Representations

Assume that $x[n]$ represents "samples" of $x(t)$:



$$x(t) = A\cos(\omega t)$$

$$x[n] = A\cos(\Omega n)$$

- What are the units of ω, t, Ω, and n?

Let f represent the "frequency" of the tone in cycles/second.

- Determine ω in terms of f.
- Determine Ω in terms of ω.
- Determine Ω in terms of f.

Check Yourself

Compare two signals:

$$x_1[n] = \cos\frac{3\pi n}{4}$$

$$x_2[n] = \cos\frac{5\pi n}{4}$$

How many of the following statements are true?

$x_1[n]$ has period N=8.

$x_2[n]$ has period N=8.

$x_1[n] = x_2[n]$.

Frequencies

Consider the following CT signal:

$$f(t) = 6\cos(42\pi t) + 4\cos(18\pi t - 0.5\pi)$$

What is the fundamental period of this signal?

Frequencies

Now imagine that this same signal

$$f(t) = 6\cos(42\pi t) + 4\cos(18\pi t - 0.5\pi)$$

is sampled with a sampling rate of $f_s = 60\,\text{Hz}$ to obtain a discrete-time signal $f[n]$, which is periodic in n with fundamental period N.

Determine the DT frequency components of $f[n]$.

Frequencies

The DT signal

$$f[n] = 6\cos(42\pi\frac{n}{60}) + 4\cos(18\pi\frac{n}{60} - 0.5\pi)$$

has a fundamental period of $N = 20$. However, this signal is also periodic in $N = 80$.

Which discrete frequencies are present if we reanalyze with $N = 80$?

Tones in Python

Determine EXPR1 and EXPR2 below to generate a 1000 Hz cosine tone using a sampling rate of $44,100$ samples/second. The tone should last 2.5 seconds.

```
import math

from lib6003.audio import wav_write

f = [cos(EXPR1 * n) for n in range(0, EXPR2)]

wav_write(f, 44100, 'output.wav')
```