6.003: Signal Processing

Fourier Series – Complex Form

Synthesis Equation (making a signal from components):

$$f(t) = f(t+T) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$$

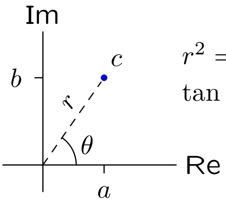
Analysis Equation (finding the components)

$$a_k = \frac{1}{T}\int_T f(t) e^{-jk\omega_o t} dt$$

where $\omega_o = \frac{2\pi}{T}$

Representations of Complex Numbers

Let c represent a complex number.



$$r^2 = a^2 + b^2$$
$$\tan\theta = \frac{b}{a}$$

rectangular form: $c = a + jb$

polar (phasor) form: $r\angle\theta$

Euler form: $r\,e^{j\theta}$

Find

$$\angle(jc) - \angle(c)$$

which can also be written as

$$\arg(jc) - \arg(c)$$

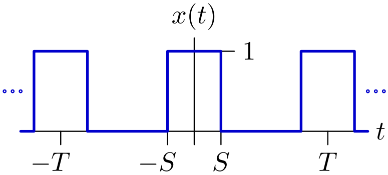
Complex Numbers

How many of the following are true?

- $$\frac{1}{\cos\theta + j\sin\theta} = \cos\theta - j\sin\theta$$
- $$(\cos\theta + j\sin\theta)^n = \cos(n\theta) + j\sin(n\theta)$$
- $$\left|2 + j2 + e^{\frac{j\pi}{4}}\right| = |2 + j2| + \left|e^{\frac{j\pi}{4}}\right|$$
- $$\operatorname{Im}\left(j^j\right) > \operatorname{Re}\left(j^j\right)$$
- $$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1} 1$$

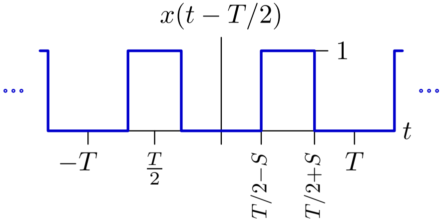
Pulse Train

Find the Fourier series coefficients a_k for $x(t)$:



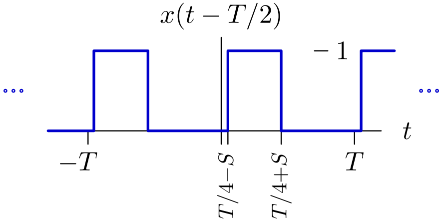
Pulse Train

What would happen to Fourier series if you delayed $x(t)$ by $T/2$?



Pulse Train

What would happen if you delayed $x(t)$ by $T/4$?



Parseval's Theorem

Determine an expression for

$$\int_T (f(t))^2 dt$$

in terms of the Fourier series coefficients a_k of $f(t)$.

$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$$

Fourier Series Matching

Match the signals (left column) to Fourier series coefficients (right).