6.003: Signal Processing

Sinusoids and Fourier Series

September 14, 2021

Fourier Series (Trigonometric Form)

If $f(t)$ is expressed as a Fourier series

$$f(t) = f(t+T) = c_0 + \sum_{k=1}^{\infty} \Big(c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t) \Big)$$

the Fourier coefficients are given by

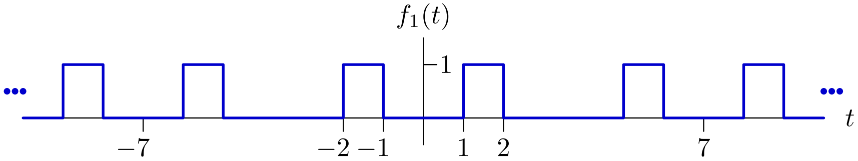
$$c_0 = \frac{1}{T} \int_T f(t)\, dt$$

$$c_k = \frac{2}{T} \int_T f(t) \cos(k\omega_o t)\, dt; \;\; k = 1, 2, 3, \ldots$$

$$d_k = \frac{2}{T} \int_T f(t) \sin(k\omega_o t)\, dt; \;\; k = 1, 2, 3, \ldots$$

Two Pulses

Let $f_1(t)$ represent the following function, which is periodic in $T = 7$:

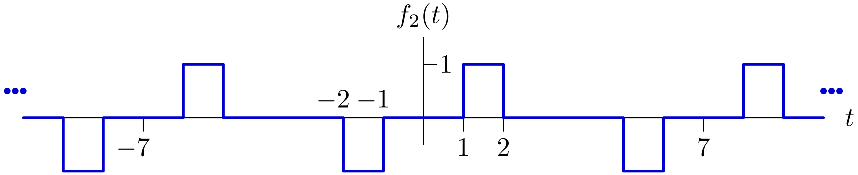


Determine a Fourier series of the following form for $f_1(t)$.

$$f_1(t) = c_0 + \sum_{k=1}^{\infty} \Big(c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t) \Big)$$

Opposite Pulses

Let $f_2(t)$ represent the following function, which is periodic in $T = 7$:

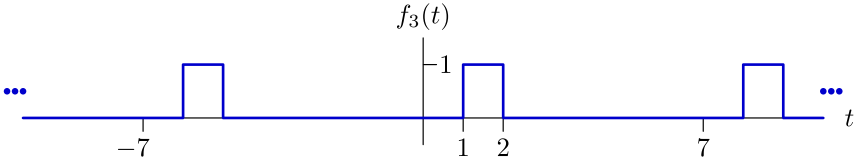


Find ω_o and the Fourier series coefficients c_k and d_k so that

$$f_2(t) = \sum_{k=0}^{\infty} \Big(c_k \cos(k\omega_o t) + d_k \sin(k\omega_o t) \Big)$$

Single Pulse

Let $f_3(t)$ represent the following function, which is periodic in $T = 7$:



Determine the Fourier series coefficients for $f_3(t)$.

Discuss the relation(s) among the Fourier series coefficients of $f_1(t)$, $f_2(t)$, and $f_3(t)$.

Discuss the relation(s) among $f_1(t)$, $f_2(t)$, and $f_3(t)$.

Trig Table

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sin(a+b) = sin(a) cos(b) + cos(a) sin(b)
sin(a-b) = sin(a) cos(b) - cos(a) sin(b)
cos(a+b) = cos(a) cos(b) - sin(a) sin(b)
cos(a-b) = cos(a) cos(b) + sin(a) sin(b)
tan(a+b) = (tan(a)+tan(b))/(1-tan(a) tan(b))
tan(a-b) = (tan(a)-tan(b))/(1+tan(a) tan(b))

sin(A) + sin(B) =  2 sin((A+B)/2) cos((A-B)/2)
sin(A) - sin(B) =  2 cos((A+B)/2) sin((A-B)/2)
cos(A) + cos(B) =  2 cos((A+B)/2) cos((A-B)/2)
cos(A) - cos(B) = -2 sin((A+B)/2) sin((A-B)/2)

sin(a+b) + sin(a-b) =  2 sin(a) cos(b)
sin(a+b) - sin(a-b) =  2 cos(a) sin(b)
cos(a+b) + cos(a-b) =  2 cos(a) cos(b)
cos(a+b) - cos(a-b) = -2 sin(a) sin(b)

2 cos(A) cos(B) = cos(A-B) + cos(A+B)
2 sin(A) sin(B) = cos(A-B) - cos(A+B)
2 sin(A) cos(B) = sin(A+B) + sin(A-B)
2 cos(A) sin(B) = sin(A+B) - sin(A-B)
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