

6.003: Signal Processing

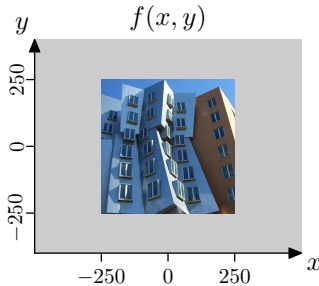
Fourier Series (Trigonometric Form)

Representing Signals as Fourier Series

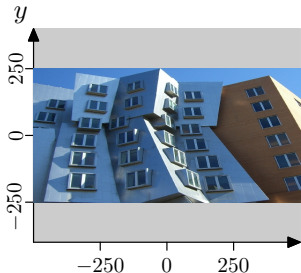
- Synthesis: making a signal from components
- Analysis: finding the components

September 9, 2021

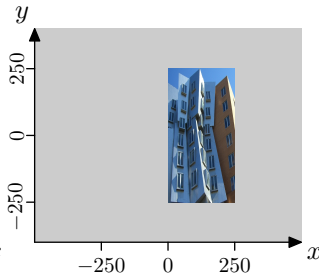
Start With Some Basic Transformations



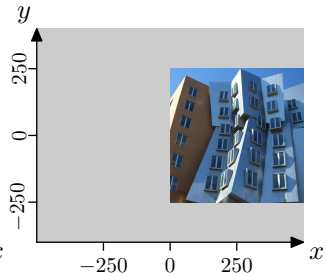
How many images match the expressions beneath them?



$$f_1(x, y) = f(2x, y) ?$$



$$f_2(x, y) = f(2x - 250, y) ?$$



$$f_3(x, y) = f(-x - 250, y) ?$$

Fourier Series

Fourier representations are a major theme of this subject.

The basic ideas were described in lecture:

Synthesis Equation (making a signal from components):

$$f(t) = f(t + T) = c_0 + \sum_{k=1}^{\infty} c_k \cos\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{\infty} d_k \sin\left(\frac{2\pi kt}{T}\right)$$

Analysis Equations (finding the components):

$$c_0 = \frac{1}{T} \int_T f(t) dt$$

$$c_k = \frac{2}{T} \int_T f(t) \cos\left(\frac{2\pi kt}{T}\right) dt ; \quad k \geq 1$$

$$d_k = \frac{2}{T} \int_T f(t) \sin\left(\frac{2\pi kt}{T}\right) dt ; \quad k \geq 1$$

Warm Up

Find the Fourier series coefficients (c_k and d_k) for

$$f(t) = \cos(t)$$

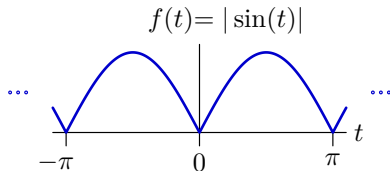
Fourier Series Coefficients

How many of the following functions have **exactly one** non-zero Fourier series coefficient?

- $f_1(t) = \cos^2 t$
- $f_2(t) = \sin t \cos t$
- $f_3(t) = 4 \cos^3 t - 3 \cos t$
- $f_4(t) = \cos(12t) \cos(4t) \cos(2t)$

Rectified Sine Wave

Consider a Fourier series representation of the following function.



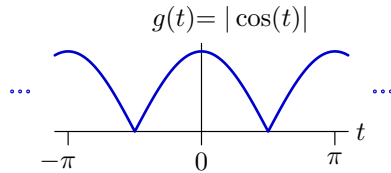
- What is the approximate value of c_0 ?
- Which non-DC Fourier coefficient has the largest absolute value? What's the sign of that coefficient?
- Determine an expression for the Fourier coefficients of $f(t)$.
- Compute the sum of the first 100 terms in the Fourier series of $f(t)$.

Verify Fourier Series of Rectified Sine Wave Numerically

Compute the sum of the first 100 terms in the Fourier series of $f(t)$.

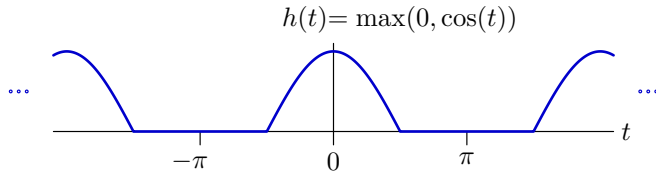
Rectified Cosine Wave

Determine the Fourier series representation of a rectified cosine.



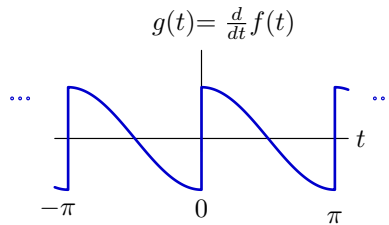
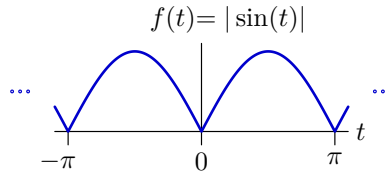
Half-Wave Rectified Cosine

Determine the Fourier series representation of a half-wave rectified cosine.



Derivative of Rectified Sine

Similarly analyze the derivative of the rectified sine with respect to time.



Verify Derivative of Rectified Sine Numerically

Compute the sum of the first 100 terms in the Fourier series of $g(t)$.

Trig Table

$$\sin(a+b) = \sin(a) \cos(b) + \cos(a) \sin(b)$$

$$\sin(a-b) = \sin(a) \cos(b) - \cos(a) \sin(b)$$

$$\cos(a+b) = \cos(a) \cos(b) - \sin(a) \sin(b)$$

$$\cos(a-b) = \cos(a) \cos(b) + \sin(a) \sin(b)$$

$$\tan(a+b) = (\tan(a)+\tan(b))/(1-\tan(a) \tan(b))$$

$$\tan(a-b) = (\tan(a)-\tan(b))/(1+\tan(a) \tan(b))$$

$$\sin(A) + \sin(B) = 2 \sin((A+B)/2) \cos((A-B)/2)$$

$$\sin(A) - \sin(B) = 2 \cos((A+B)/2) \sin((A-B)/2)$$

$$\cos(A) + \cos(B) = 2 \cos((A+B)/2) \cos((A-B)/2)$$

$$\cos(A) - \cos(B) = -2 \sin((A+B)/2) \sin((A-B)/2)$$

$$\sin(a+b) + \sin(a-b) = 2 \sin(a) \cos(b)$$

$$\sin(a+b) - \sin(a-b) = 2 \cos(a) \sin(b)$$

$$\cos(a+b) + \cos(a-b) = 2 \cos(a) \cos(b)$$

$$\cos(a+b) - \cos(a-b) = -2 \sin(a) \sin(b)$$

$$2 \cos(A) \cos(B) = \cos(A-B) + \cos(A+B)$$

$$2 \sin(A) \sin(B) = \cos(A-B) - \cos(A+B)$$

$$2 \sin(A) \cos(B) = \sin(A+B) + \sin(A-B)$$

$$2 \cos(A) \sin(B) = \sin(A+B) - \sin(A-B)$$