

6.003 Quiz 2

Spring 2020

Name:

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use one 8.5×11 sheet of paper (two sides).
You may NOT use any electronic devices (including calculators, phones, etc).

If you have questions, please **come to us at the front** to ask them.

Please enter all solutions in the boxes provided.

Extra work may be taken into account when assigning partial credit,
but only work on pages with QR codes will be considered.

Question 1: 20 Points

Question 2: 24 Points

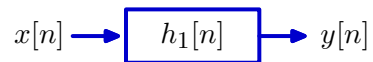
Question 3: 24 Points

Question 4: 32 Points

Total: 100 Points

1 Systems

Part 1. Let $h_1[n]$ represent the unit-sample response of a discrete-time, linear, time-invariant system.



If the input signal is

$$x[n] = \delta[n] + \delta[n-1]$$

then the output signal is

$$y[n] = \delta[n] + 2\delta[n-1] + 2\delta[n-2] + \delta[n-3].$$

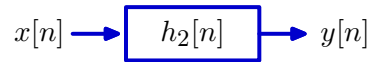
Find the unit-sample response $h_1[n]$, assuming that $h_1[n] = 0$ for $n < 0$.

Enter the first 5 samples of your result as a python list

$$[h_1[0], h_1[1], h_1[2], h_1[3], h_1[4]]$$

in the box below.

Part 2. Let $h_2[n]$ represent the unit-sample response of a discrete-time, linear, time-invariant system.



The input signal $x[n]$ and output signal $y[n]$ are related by the following difference equation for all n .

$$y[n+1] = \frac{1}{2}(x[n] + y[n])$$

Part 2a. Determine the unit-sample response $h_2[n]$ of this system, assuming that $h_2[n] = 0$ for $n < 0$.

Enter the first 5 samples of your result as a python list

[$h_2[0]$, $h_2[1]$, $h_2[2]$, $h_2[3]$, $h_2[4]$]

in the box below.

Part 2b. Determine the frequency response of this system.

Enter an expression for $H_2(\Omega)$ in the box below.

Part 3. Let

$$x[n] = \begin{cases} 1 & 0 \leq n \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

and let $X[k]$ represent the DFT of $x[n]$ with analysis period $N = 8$.

Part 3a. Find $X[0]$ and $X[1]$ and enter them (as numerical expressions) in the boxes below.

$X[0] =$

$X[1] =$

Part 3b. Let $Y[k] = X^2[k]$. Determine $y[n]$ which represents the inverse DFT of $Y[k]$. Enter your result as a python list of the form

[$y[0]$, $y[1]$, $y[2]$, ...]

2 Peaks and Valleys

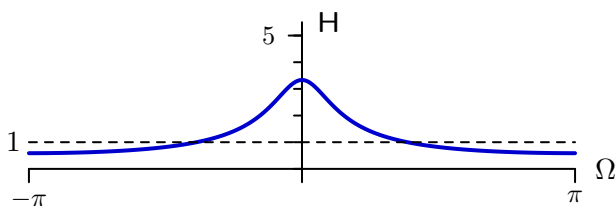
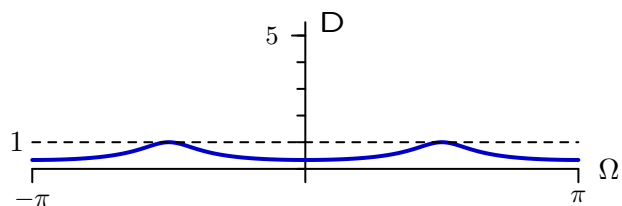
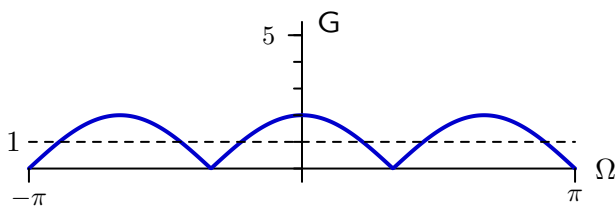
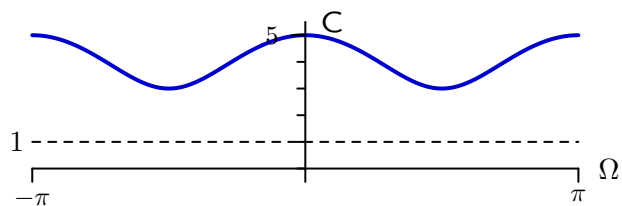
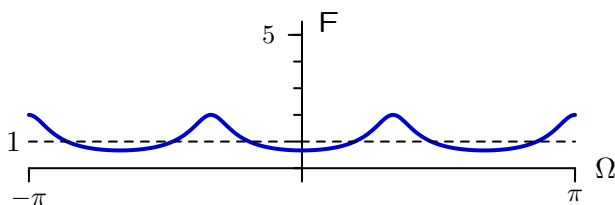
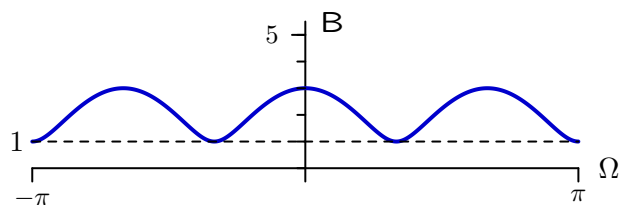
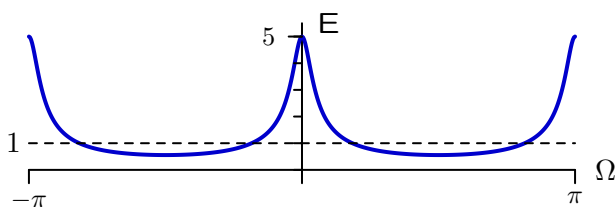
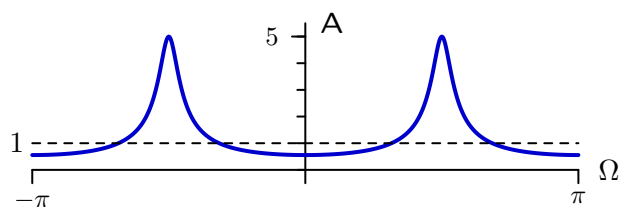
Each of the plots below shows the magnitude of the frequency response of a discrete-time system that can be described by the following difference equation, where α and m are parameters.

$$y[n] = x[n] + \alpha y[n-m]$$

Each row in the following table gives the parameters for one of the plots.

Write the letter of the corresponding plot in the right column of the table.

α	m	Frequency Response Enter A–H
0.7	1	
-0.5	3	
-0.8	2	
-2	2	
0.8	2	



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3 Composite Systems

In this problem, we consider eight linear, time-invariant systems whose unit-sample responses are expressed in terms of $g_1[n]$ and $g_2[n]$ where

$$g_1[n] = \delta[n] - \delta[n-1] + \delta[n-2] \text{ and}$$

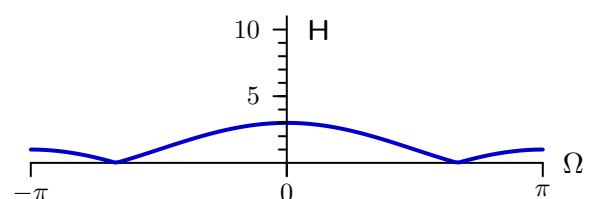
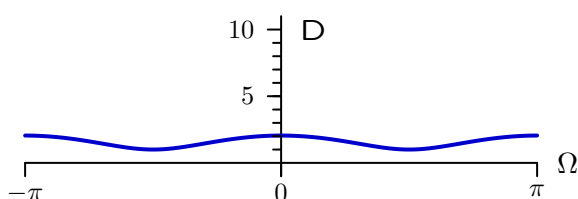
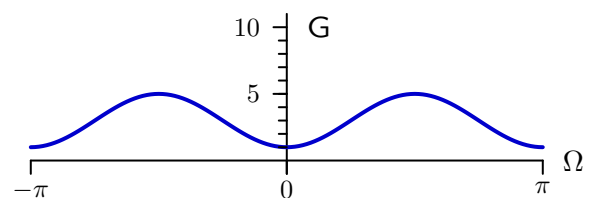
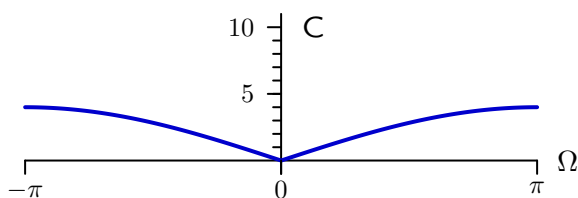
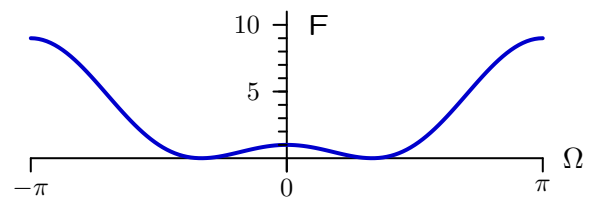
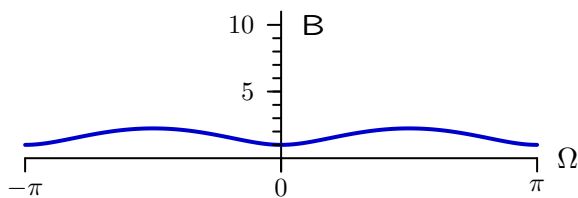
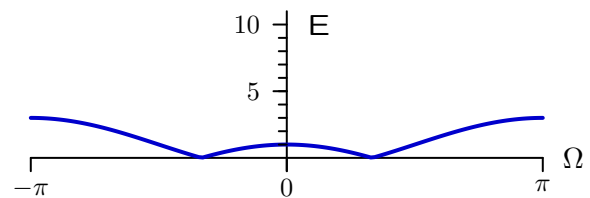
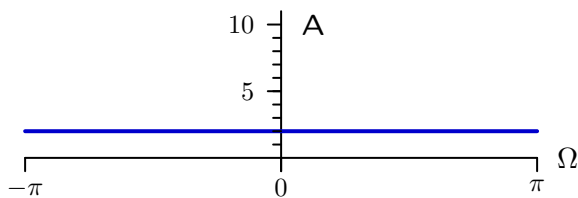
$$g_2[n] = \delta[n] + \delta[n-1] - \delta[n-2].$$

The unit-sample response $h_i[n]$ of each system is given by the expression in the center column of the table below.

System	Unit-sample response	Frequency Response Enter A–H
1	$h_1[n] = g_1[n]$	
2	$h_2[n] = g_2[n]$	
3	$h_3[n] = g_1[n] + g_2[n]$	
4	$h_4[n] = g_1[n] - g_2[n]$	
5	$h_5[n] = g_1[n] \times g_1[n]$	
6	$h_6[n] = g_1[n] \times g_2[n]$	
7	$h_7[n] = (g_1 * g_1)[n]$	
8	$h_8[n] = (g_2 * g_2)[n]$	

The **magnitude** of the frequency response for each of systems 1–8 is shown by one of the plots A–H below.

Determine which applies and enter the appropriate letter (A–H) in the right column. Answers may be repeated.



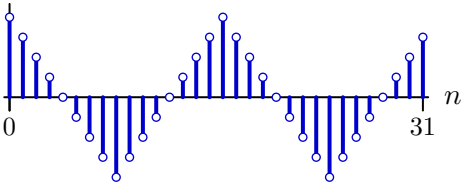
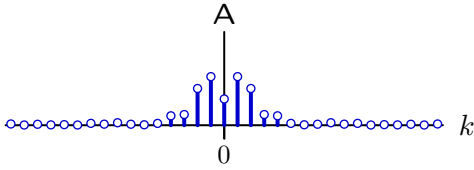
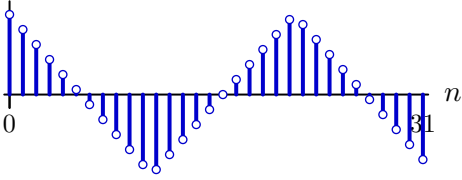
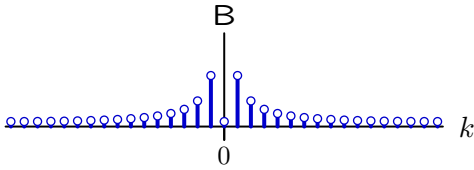
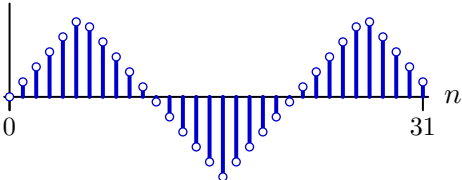
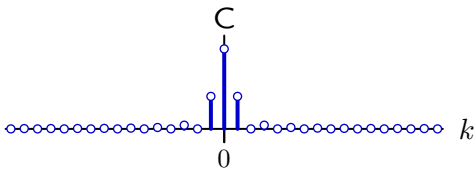
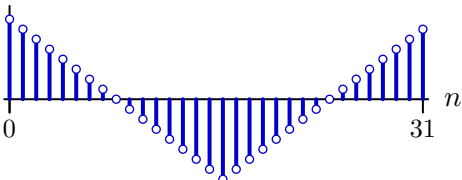
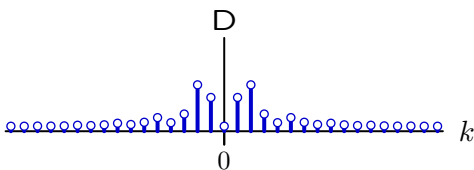
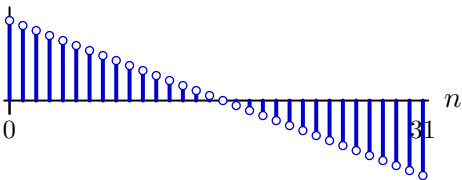
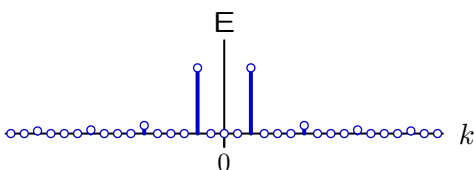
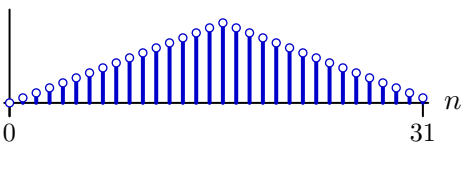
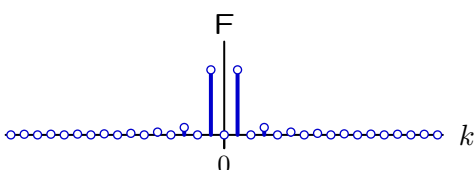
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4 Discrete Fourier Transforms

The left column below shows six discrete-time signals for $0 \leq n \leq 31$.

The right column shows plots of the magnitudes of six DFTs computed for $N = 32$.

For each discrete-time signal in the left column below, find the matching DFT magnitude (one of plots A-F) and enter its letter in the box provided.

DT signals	Corresponding DFT magnitude plot (A-F)	plots
	$\xLeftrightarrow{\text{DFT}}$ <input style="width: 50px; height: 30px; border: 1px solid black;" type="text"/>	<p>A</p> 
	$\xLeftrightarrow{\text{DFT}}$ <input style="width: 50px; height: 30px; border: 1px solid black;" type="text"/>	<p>B</p> 
	$\xLeftrightarrow{\text{DFT}}$ <input style="width: 50px; height: 30px; border: 1px solid black;" type="text"/>	<p>C</p> 
	$\xLeftrightarrow{\text{DFT}}$ <input style="width: 50px; height: 30px; border: 1px solid black;" type="text"/>	<p>D</p> 
	$\xLeftrightarrow{\text{DFT}}$ <input style="width: 50px; height: 30px; border: 1px solid black;" type="text"/>	<p>E</p> 
	$\xLeftrightarrow{\text{DFT}}$ <input style="width: 50px; height: 30px; border: 1px solid black;" type="text"/>	<p>F</p> 

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