

6.003 Quiz 1

Spring 2020

Name:

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use one 8.5×11 sheet of paper (two sides).
You may NOT use any electronic devices (including calculators, phones, etc).

If you have questions, please **come to us at the front** to ask them.

Please enter all solutions in the boxes provided.

Extra work may be taken into account when assigning partial credit,
but only work on pages with QR codes will be considered.

Question 1: 20 Points

Question 2: 24 Points

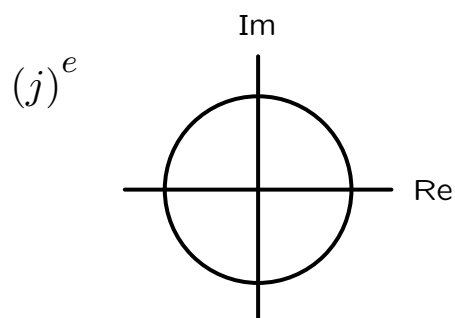
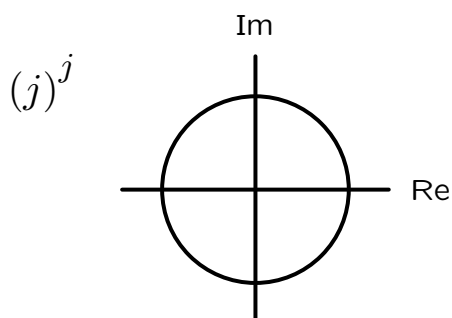
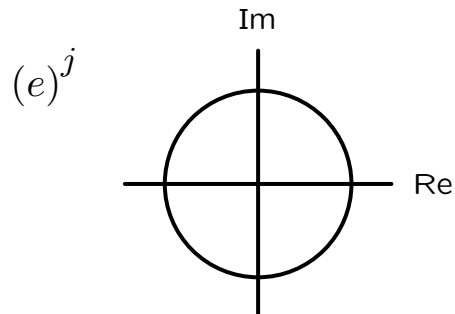
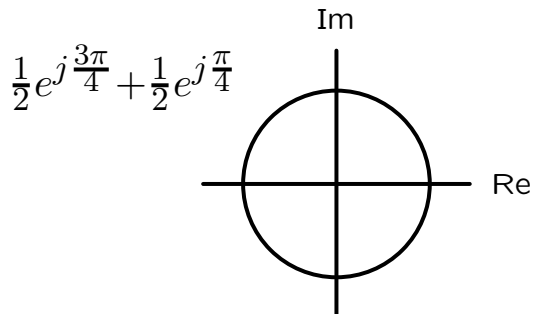
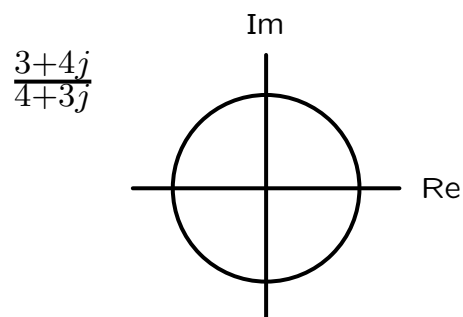
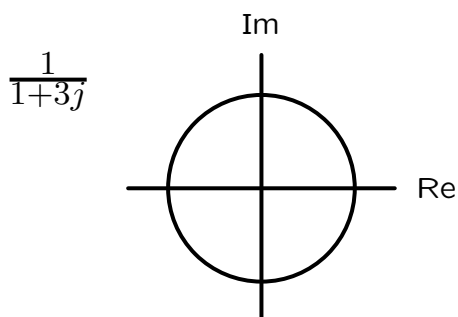
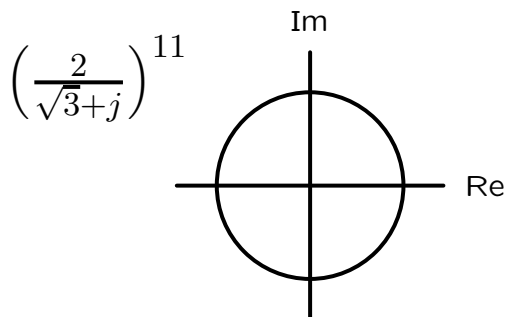
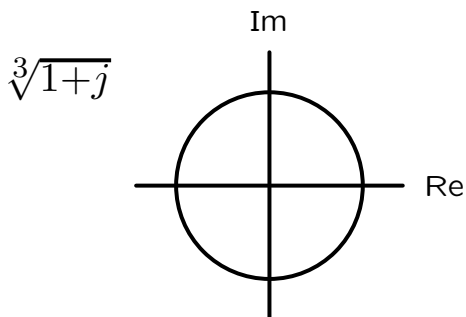
Question 3: 24 Points

Question 4: 32 Points

Total: 100 Points

1 Plainly Complex (20 Points)

Below are eight complex-valued expressions, each paired with a depiction of the complex plane demarcated by the unit circle. Evaluate each expression and mark its value on the complex plane with a dot. If the expression can represent multiple complex numbers, mark at least two of them.



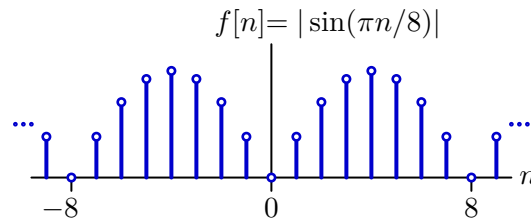
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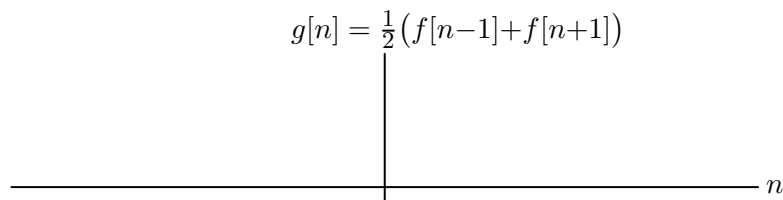
3 Rectified Sine Wave (24 Points)

Part a. Let $f[n]$ represent the following periodic, discrete-time signal:



Let $g[n] = \frac{1}{2}(f[n-1] + f[n+1])$.

Sketch $g[n]$ on the following axes. Label the important parameters of your plot.



Let $F[k]$ represent the Fourier series coefficients for $f[n]$ computed with **period $N = 8$** :

$$F[k] = \frac{1}{N} \sum_{n=\langle N \rangle} f[n] e^{-j \frac{2\pi k}{N} n}$$

Let $G[k]$ represent the Fourier series coefficients for $g[n]$ computed with **same period $N = 8$** .

Determine the relation between the $G[k]$ coefficients and the $F[k]$ coefficients.

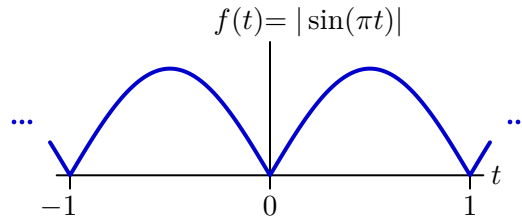
In the table below, enter an expression for each of $G[0]$ through $G[7]$ in terms of $F[0], F[1], F[2], \dots$

In addition to $F[0], F[1], F[2], \dots$, your table entries can contain real and/or imaginary numbers and constants such as e and π . Your entries should not contain integrals or summations.

k	$G[k]$
0	
1	
2	
3	
4	
5	
6	
7	

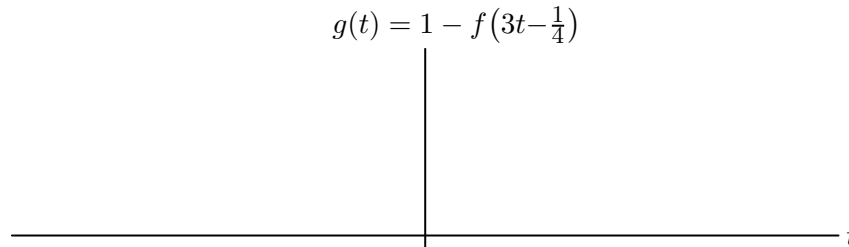
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Part b. Let $f(t)$ represent the following periodic, continuous-time signal:



Let $g(t) = 1 - f(3t - \frac{1}{4})$.

Sketch $g(t)$ on the following axes. Label the important parameters of your plot.



Let $F[k]$ represent the Fourier series coefficients for $f(t)$ computed with **period $T = 1$** :

$$F[k] = \frac{1}{T} \int_T f(t) e^{-j \frac{2\pi k}{T} t} dt$$

Let $G[k]$ represent the Fourier series coefficients for $g(t)$ computed with **same period $T = 1$** .

Determine the relation between the $G[k]$ coefficients and the $F[k]$ coefficients.

In the tables below, enter expressions for each of $G[0]$ through $G[15]$ in terms of $F[0]$, $F[1]$, $F[2]$, \dots

In addition to $F[0]$, $F[1]$, $F[2]$, \dots , your table entries can contain real and/or imaginary numbers and constants such as e and π . Your equations should not contain integrals or summations.

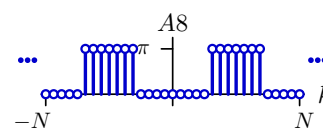
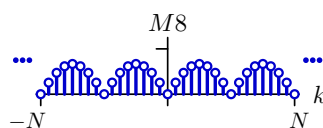
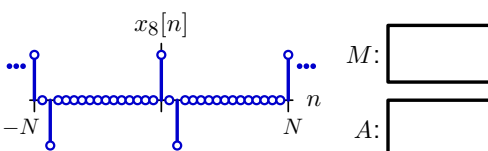
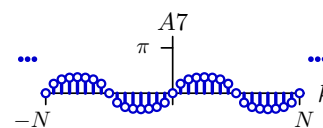
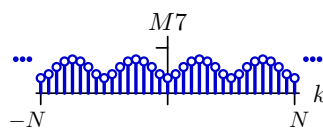
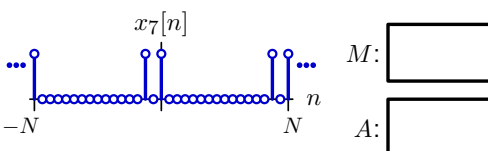
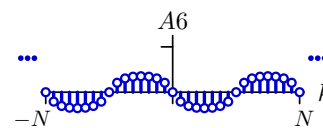
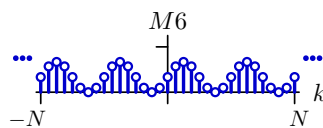
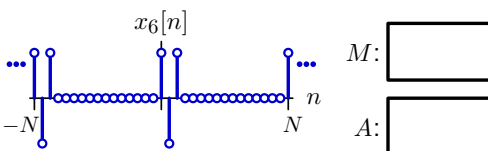
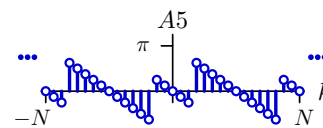
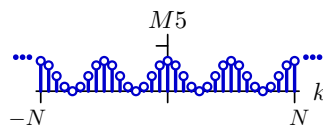
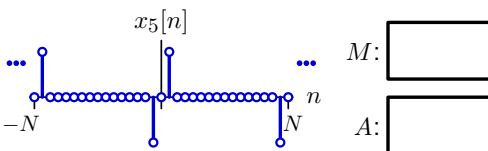
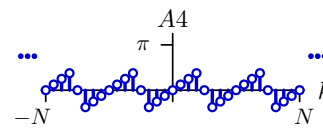
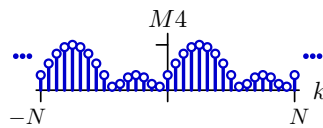
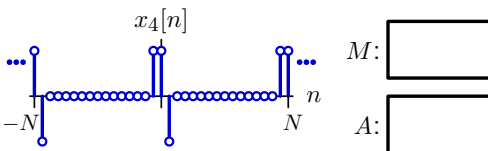
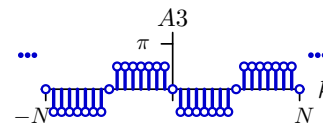
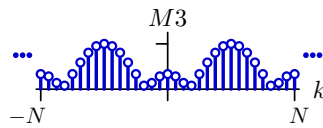
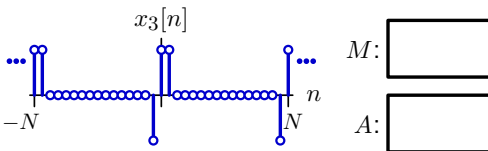
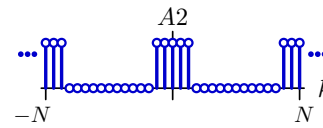
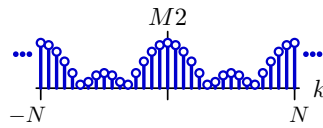
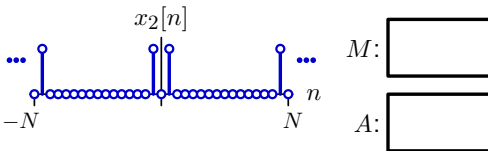
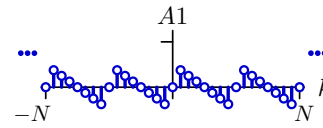
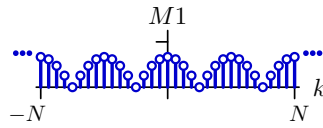
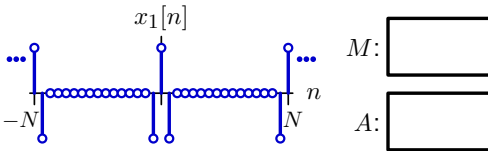
k	$G[k]$
0	
1	
2	
3	
4	
5	
6	
7	

k	$G[k]$
8	
9	
10	
11	
12	
13	
14	
15	

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4 Fourier Series Matching (32 Points)

Each of the signals $x_i[n]$ in the left column below is periodic with period $N = 16$. Find the Fourier series coefficients $X_i[k]$ for each signal and then identify which of plots $M1 - M8$ shows the magnitude of $X_i[k]$ and which of plots $A1 - A8$ shows the angle of $X_i[k]$ as functions of k . Enter your answers in the boxes provided.



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