# 6.003: Signal Processing

## Magnetic Resonance Imaging[1]

Please give us feedback on 6.003:
End-of-Term **Subject Evaluations** now till Monday, Dec 13 at 9am.

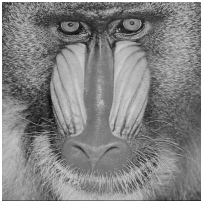
Last time to **submit homeworks** is Friday, Dec 3, 5pm.

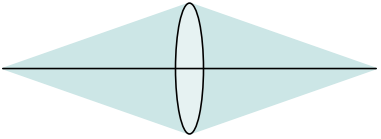
**Final Exam:** Thursday, Dec 16: 1:30-4:30pm, Dupont Gym
Conflict Exam (assigned by Registrar): Friday, Dec 17, 9:00am-noon

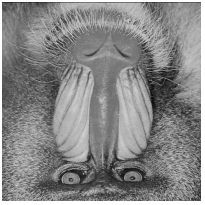
[1] Prof. Elfar Adalsteinsson provided the data and examples used here.

# Magnetic Resonance Imaging

Each pixel in a convential camera reports the amount of light at a particular position in **space.** The collection of pixels represents a spatial mapping of light intensity and produces a **image of space.**







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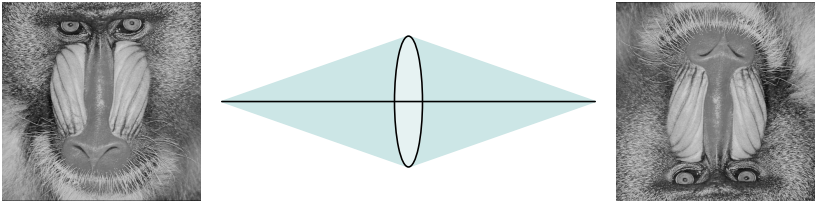
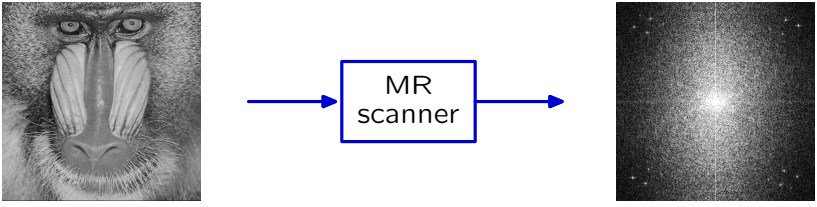


## MRI is different.

An MR scanner collects data that represent samples in **Fourier space.** The collection of measurements provides the **Fourier transform** of an image.

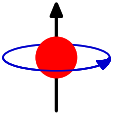


# Magnetic Resonance Imaging

Today's goal is to motivate how MRI works and why MR images are different, so that we can understand how these unique signals are processed.

# Magnetic Resonance

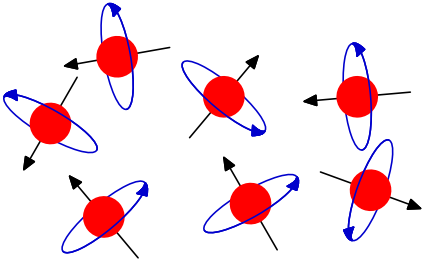
Magnetic resonance can be understood in terms of how the spin angular momentum of a hydrogen nucleus (i.e., a proton) and its associated magnetic dipole moment interact with an external magnetic field.[1]



[1] Although spin angular momentum only arises in quantum mechanics, we consider a classical model that captures many important features of magnetic resonance.

# Magnetic Resonance

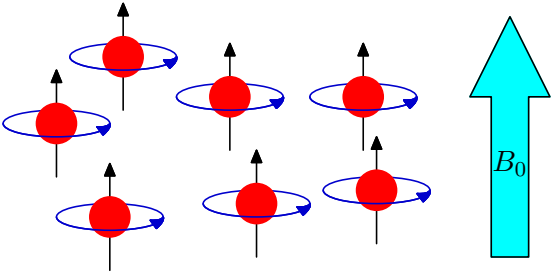
Normally, spins are randomly oriented.



There is no net magnetization.

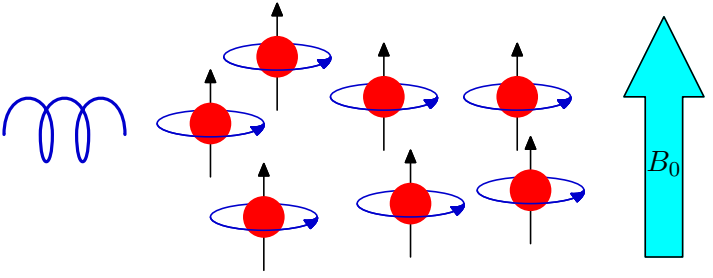
# Magnetic Resonance

Spins align with a strong (3–7 T (tesla)) external magnetic field $B_0$.



# Magnetic Resonance

Spins are tipped by brief excitation $B_1$ from a transversely oriented coil.



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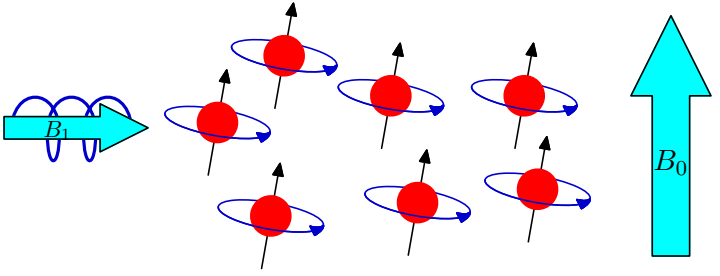
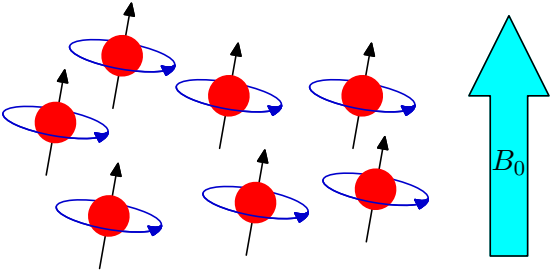


# Magnetic Resonance

The spins "precess" as they relax back to their previous alignment.




Precession frequency $\omega$ is proportional to external field strength $B$.
The constant of proportionality $\gamma = 42.58\,\mathrm{MHz/T}$ for water.

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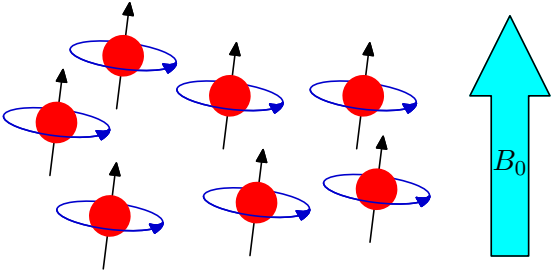


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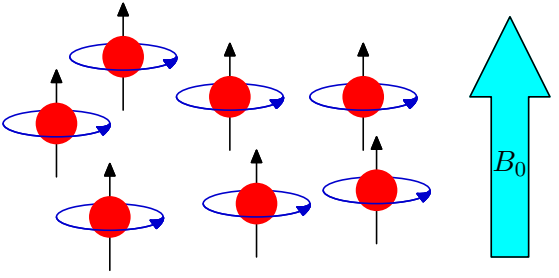


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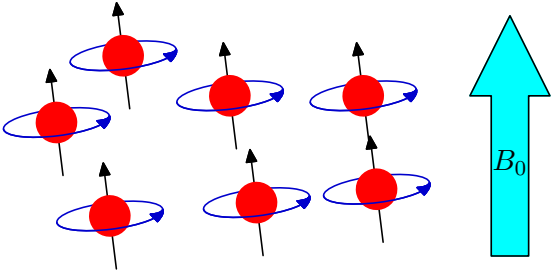


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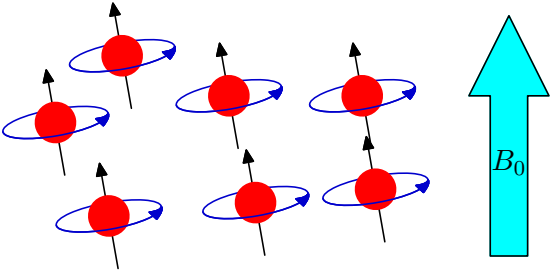


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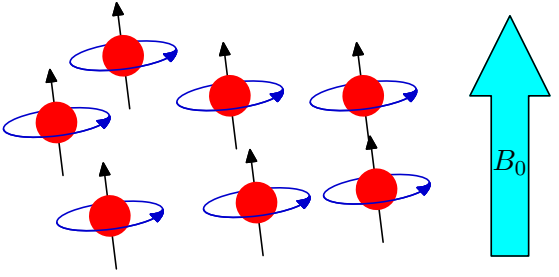


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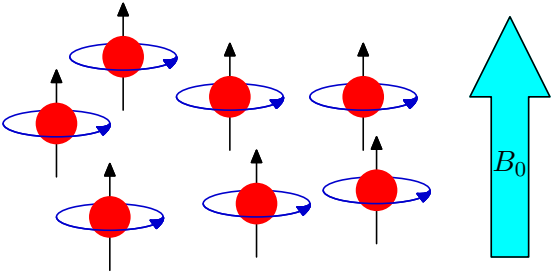


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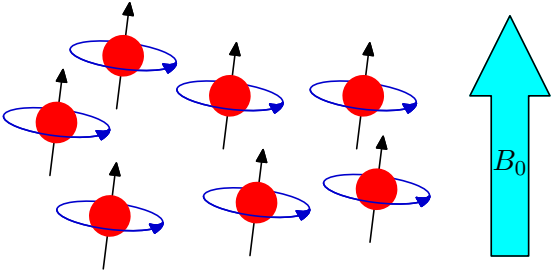


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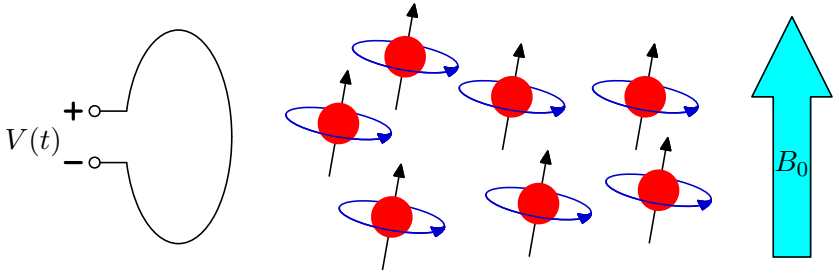
# Magnetic Resonance

Precession $\rightarrow$ changing magnetic field that is detected with a coil.



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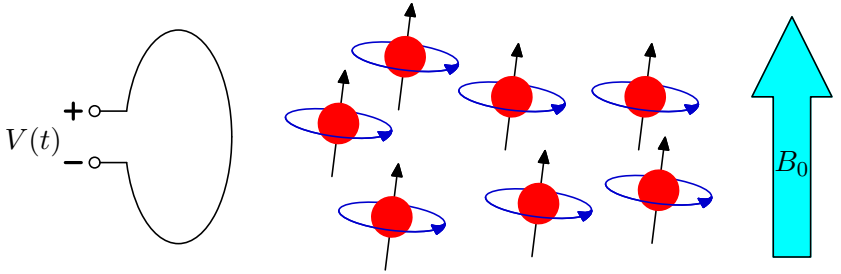

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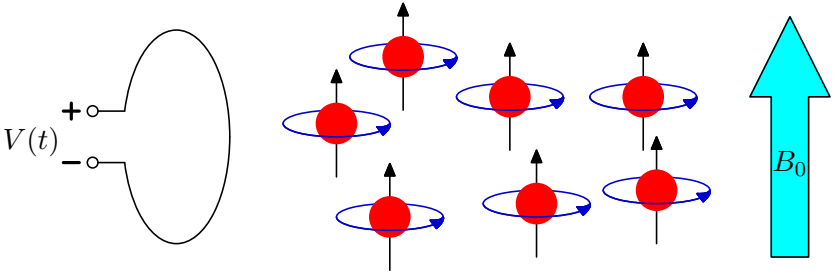

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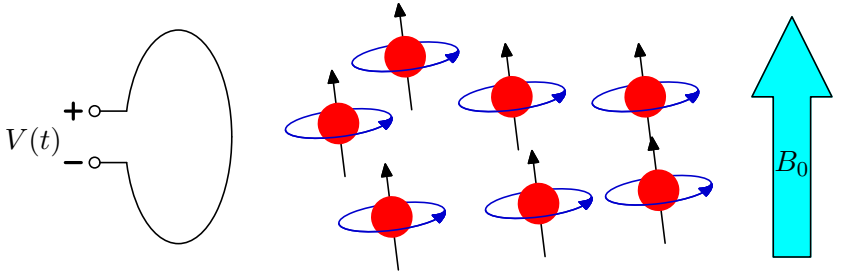

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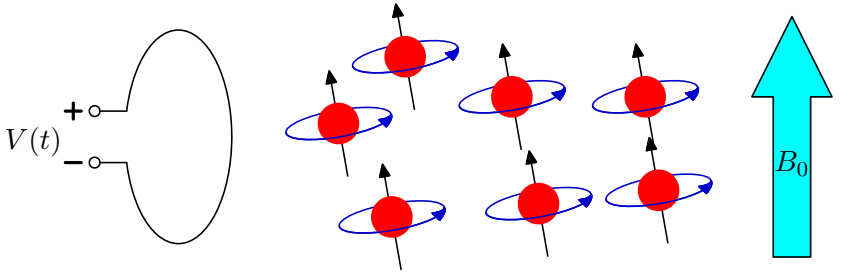

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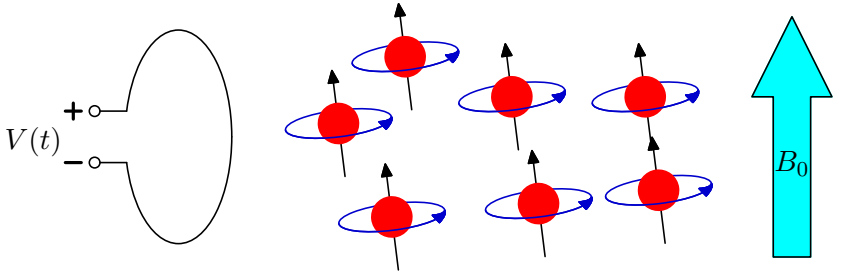

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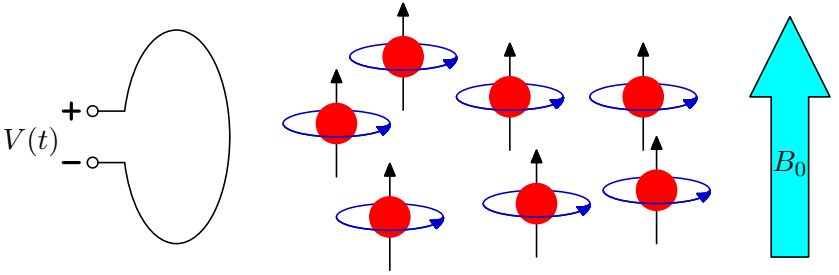

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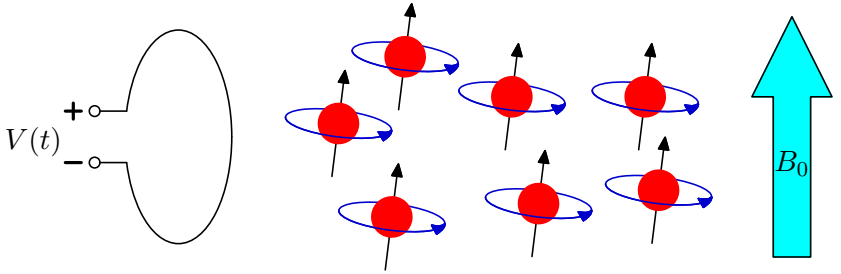
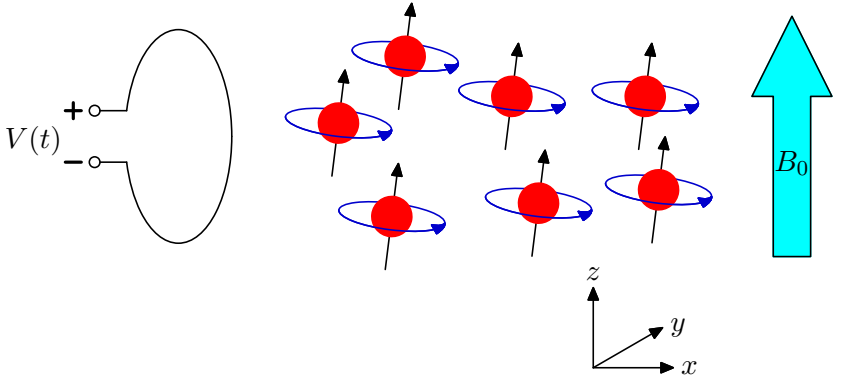


# Magnetic Resonance Imaging

The amplitude of this changing magnetic field is proportional to the number of protons in the region, and the phase of the magnetic field is the integral of the precession frequency $\omega = \gamma B_0$ with respect to time

$$\Phi(t) = \iint m(x,y) e^{-j\int_0^t \gamma B_0 d\tau}\, dx\, dy = \iint m(x,y) e^{-j\gamma B_0 t}\, dx\, dy$$

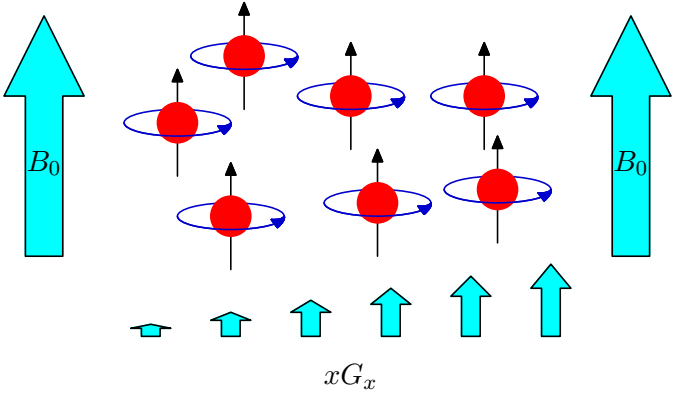
where $m(x,y)$ represents the density of protons as a function of $x$ and $y$.



# Magnetic Resonance Imaging

MRI uses the time-varying field $\Phi(t)$ to generate an image proportional to $m(x, y)$. Getting information about the spatial distribution of $m(x, y)$, requires addition of gradient fields $G_x$ and $G_y$ that vary linearly in $x$ and $y$ so that the total longitudinal field is

$$B_z(x, y)\,\hat{z} = (B_0 + xG_x + yG_y)\,\hat{z}\,.$$



# Magnetic Resonance Imaging

The spatial variations in field strength cause protons at different $x$, $y$ locations to precess at different frequencies. Their contributions to the measured field $\Phi(t)$ vary with their phase (since it's the real and imaginary parts that sum):

$$\begin{aligned}\Phi(t) &= \iint m(x,y) e^{-j\int_0^t \gamma(B_0 + xG_x + yG_y)d\tau}\, dx\, dy \\ &= \iint m(x,y) e^{-j2\pi(k_x x + k_y y)} e^{-j\gamma B_0 t}\, dx\, dy\end{aligned}$$

where

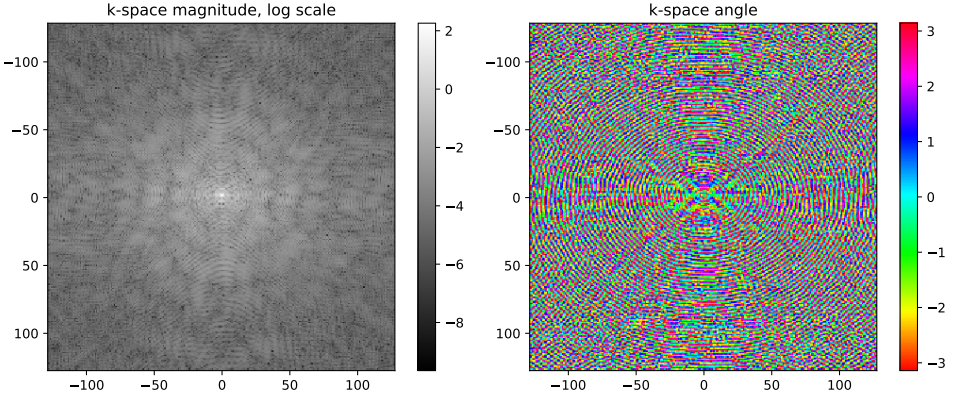
$$k_x = \frac{\gamma}{2\pi}\int_0^t G_x d\tau$$

$$k_y = \frac{\gamma}{2\pi}\int_0^t G_y d\tau$$

Samples of $\Phi(t)$ provide samples of the 2D Fourier transform of $m(x,y)$.

# Example Image

By sampling $V(t)$ as $G_x$ and $G_y$ is varied, we can assemble a $256 \times 256$ array of **k-space** data $M[k_x, k_y]$ of the following form.

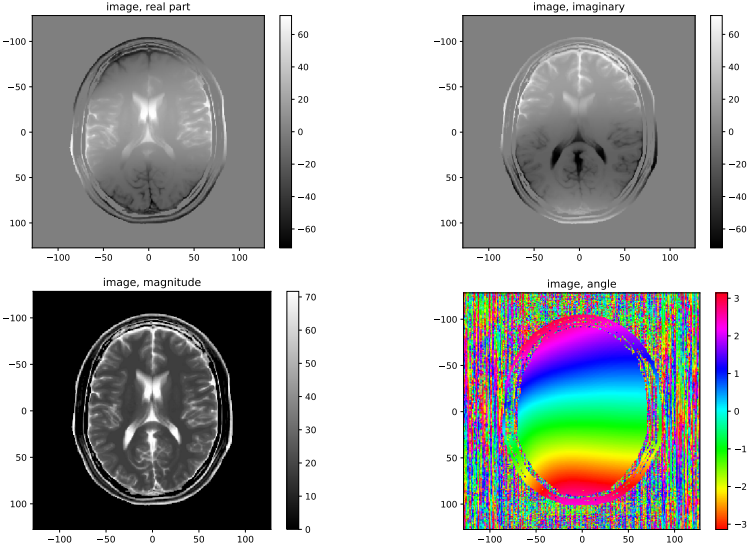



These direct measurements do NOT represent the image.

The represent the Fourier transform of the image.

# Example Image

The inverse transform of $M[k_x, k_y]$ reveals the underlying image $m[x, y]$.




The reconstructed image has both real and imaginary parts because of phase delays in the RF signal path (not considered here). As a result, the magnitude of the resulting image is a better measure of $m(x, y)$.

# Scan Time

How long does it take to obtain an image like the one on the previous slide?

Typically, one can measure an entire row or column of data as the $\int_0^t G_x d\tau$ and/or $\int_0^t G_y d\tau$ ramps up after a single RF excitation.

If RF excitation occurs once every 2 seconds, then the total acquisition time would be $256 \times 2$ seconds, which is approximately $8.5$ minutes.

This is a long time even for a healthy young adult. What about a child? Or a patient with uncontrolled tremors?

Reducing scan time is an active area of research.

## Accelerating Imaging

An important area of current research is in decreasing the time required to capture an image.

One idea for accelerating imaging is to downsample the frequency representation.

What is the effect of measuring $F[k_r, k_c]$ at only even values of $k_c$?

# Accelerating Imaging

Let $F[k_r, k_c]$ represent the original k-space data and $G[k_r, k_c]$ represent the k-space data after odd numbered columns are set to zero.

$$G[k_r, k_c] = F[k_r, k_c]\left(\frac{1 + (-1)^{k_c}}{2}\right) = \frac{1}{2}F[k_r, k_c]\left(1 + e^{j\pi k_c}\right)$$

$$\begin{aligned}
g[r, c] &= \sum_{k_r, k_c} \frac{1}{2}F[k_r, k_c]\left(1 + e^{j\pi k_c}\right) e^{j\frac{2\pi k_r}{R}r} e^{j\frac{2\pi k_c}{C}c} \\
&= \frac{1}{2}f[r, c] + \frac{1}{2}\sum_{k_r, k_c} F[k_r, k_c] e^{j\pi k_c} e^{j\frac{2\pi k_r}{R}r} e^{j\frac{2\pi k_c}{C}c} \\
&= \frac{1}{2}f[r, c] + \frac{1}{2}\sum_{k_r, k_c} F[k_r, k_c] e^{j\frac{2\pi k_r}{R}r} e^{j\frac{2\pi k_c}{C}(c + \frac{C}{2})} \\
&= \frac{1}{2}f[r, c] + \frac{1}{2}f\left[r, \left(c + \frac{1}{2}C\right) \bmod C\right]
\end{aligned}$$

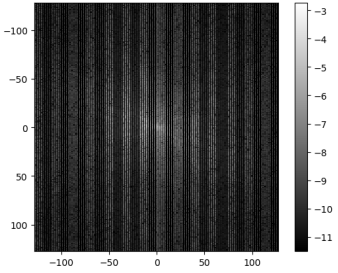
Setting odd-numbered columns of $F[k_r, k_c]$ to zero adds a half-frame circular shift to the right (or equivalently left) of the image.

# Accelerating Imaging

Omitting odd numbered columns in k-space.

$\log|G|$ $\angle(G)$

measure `G[kr,kc]`:



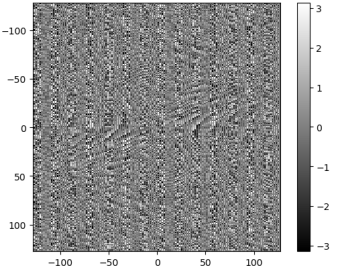
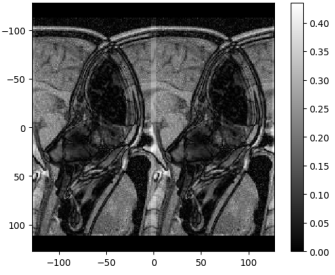



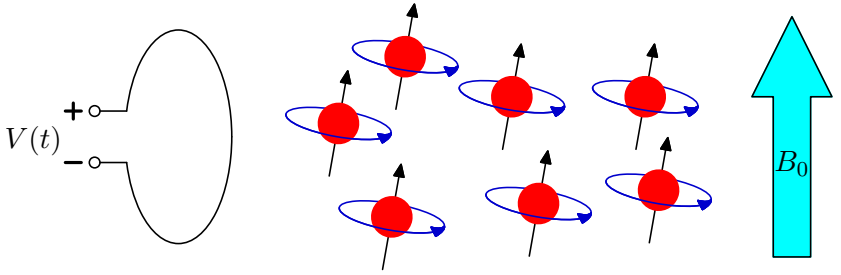

image `g = abs(ifft2(G))`:




Why is undersampling so different for MRI (compared to audio or pictures)?

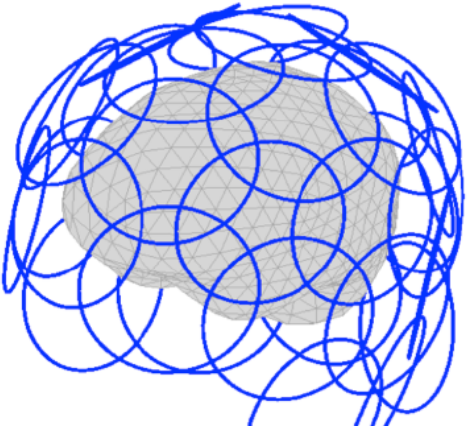
# Multi-Coil Imaging

Multiple readout coils can be read in parallel, and thereby provide additional data without increasing imaging time.



# Multi-Coil Imaging

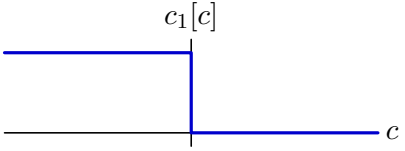
"Helmets" with as many as 16 to 32 readout coils have been used to increase the resolution of brain images.

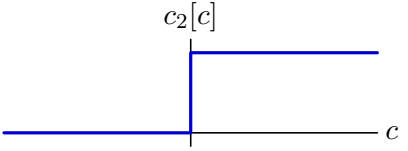


# Reconstructing Images from Multi-Coil Data

Consider two coils, one on each side of the head. The left coil will be more sensitive to the left portions of the image, and vice versa.

Characterize the **sensitivity** of each coil by specifying a a number between 0 (insensitive) and 1 (sensitive) for each pixel in the image.
In 1D, these sensitivities could have the following form:

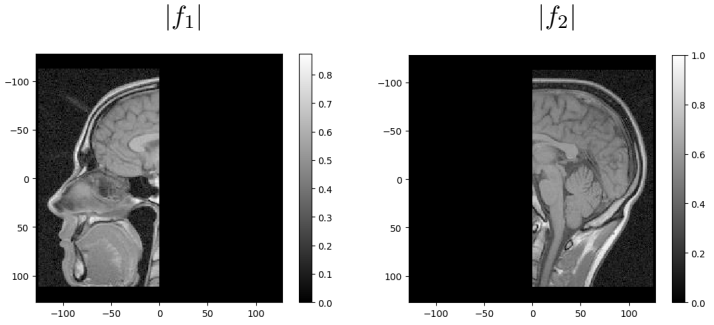



What would be the effect of these coils on the resulting image?

# Images From Coils 1 and 2

Since coil 1 is only sensitive to half of the head, the image produced with data from coil 1 shows just that half.

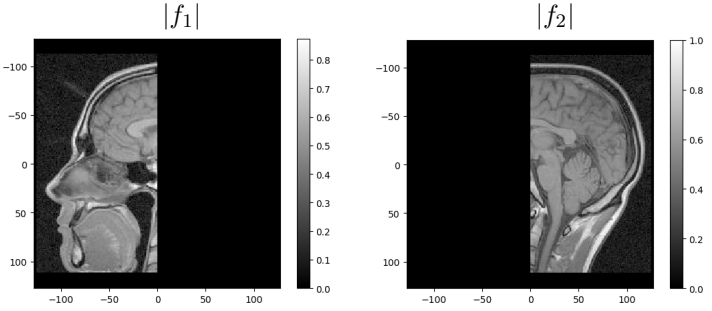



If we only measure $F_1[k_r, k_c]$ at even-numbered $k_c$, then the image from coil 1 will be added to a circularly shifted version of itself.

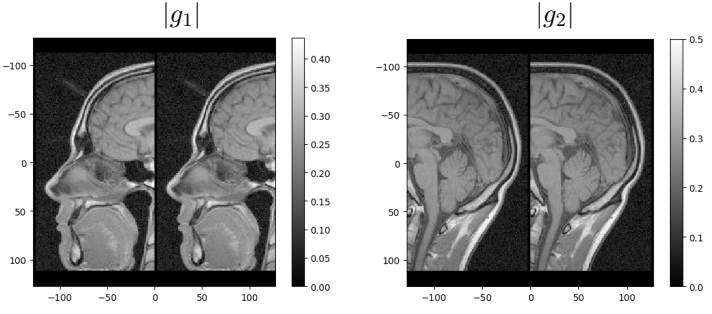
The same holds for coil 2.

# Images From Coils 1 and 2

Images $f_1$ and $f_2$ are derived from full-resolution data $F_1$ and $F_2$.




Images $g_1$ and $g_2$ are derived from just the even-numbered $k_c$.



# Images From Coils 1 and 2

Could you construct a full-frame full-resolution image from these data?

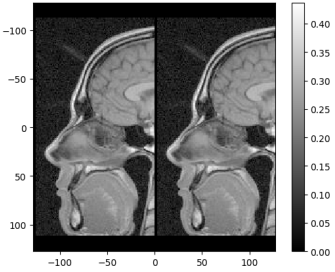
Yes. Combine the left part of $|g_1|$ with the right part of $|g_2|$.

Advantage:
$|g_1|$ was acquired in half the time required for a full-frame full-resolution image. Similar with $|g_2|$.

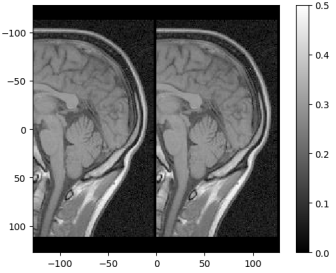
But $|g_1|$ and $|g_2|$ data were acquired simultaneously!

# Constructing Full-Frame Image From Coil 1 and 2 Data

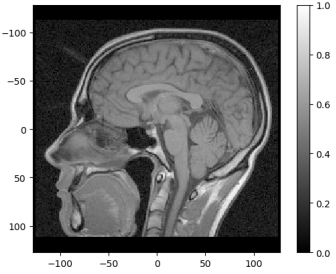
$|g_1|$




$|g_2|$

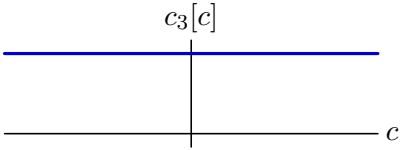


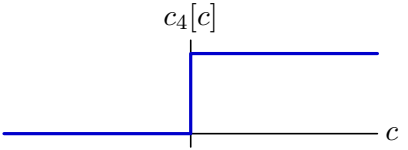

combined



# Multi-Coil MRI

What if the coils had the following sensitivities?

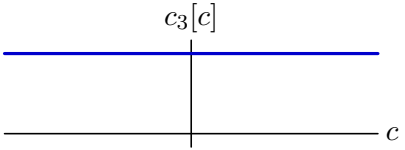


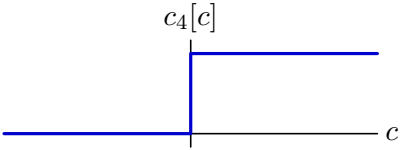




What would be the effect of each of these coils on the image?

# Multi-Coil MRI

What if the coils had the following sensitivities?

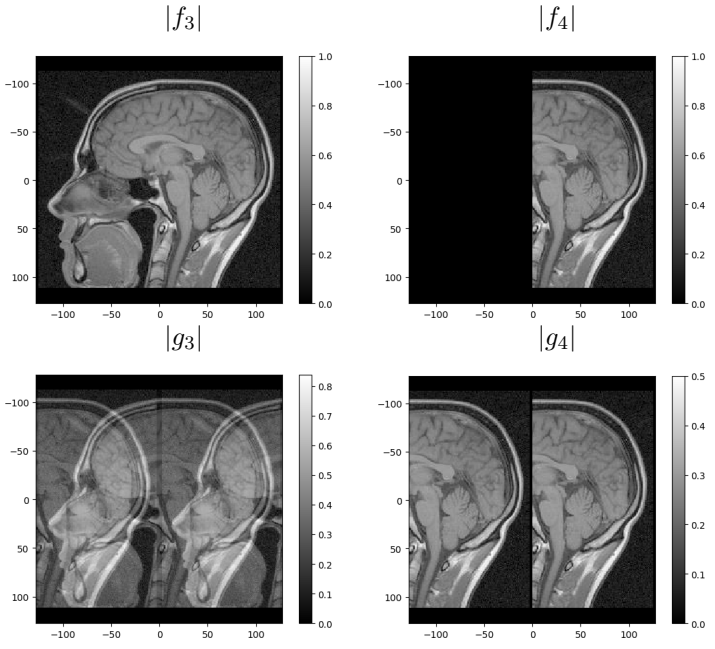



What would be the effect of each of these coils on the image?

$c_3$ is a full-frame image. Omitting the odd number columns from $G_3$ will produce the aliased image we started with.
$c_4$ is the same as the previous $c_2$, so $|g_4|$ is the same as $|g_2|$.

# Images From Coils 3 and 4




Can we create a full-frame full-resolution image from this data?

# Images From Coils 3 and 4

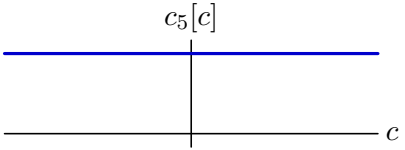
The $|f_3|$ image can be viewed as the sum of results for the left and right sides of the image (as in the $c_1$ and $c_2$ example).

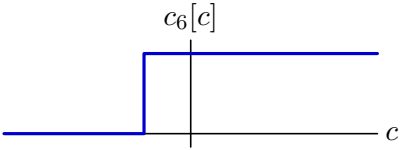
Subtracting $|g_4|$ from $|g_3|$ would generate the previous $|g_1|$ image.

Algorithm:
Combine the left part of $|g_3| - |g_4|$ with the right part of $|g_4|$.

# Multi-Coil MRI

What if the coils had the following sensitivities?

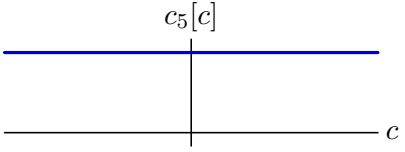


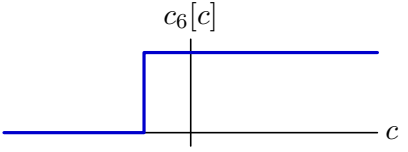




What would be the effect of each of these coils on the image?

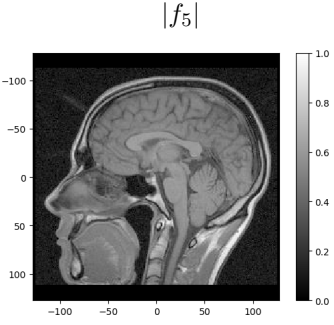
# Multi-Coil MRI

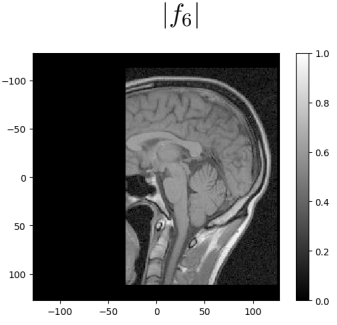
What if the coils had the following sensitivities?



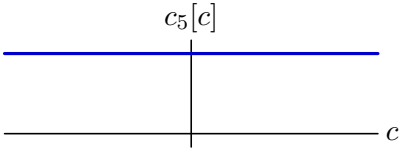

What would be the effect of each of these coils on the image?

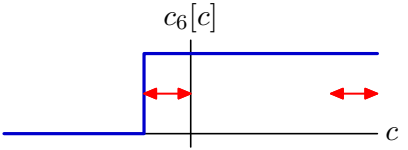


# Multi-Coil MRI

What if the coils had the following sensitivities?



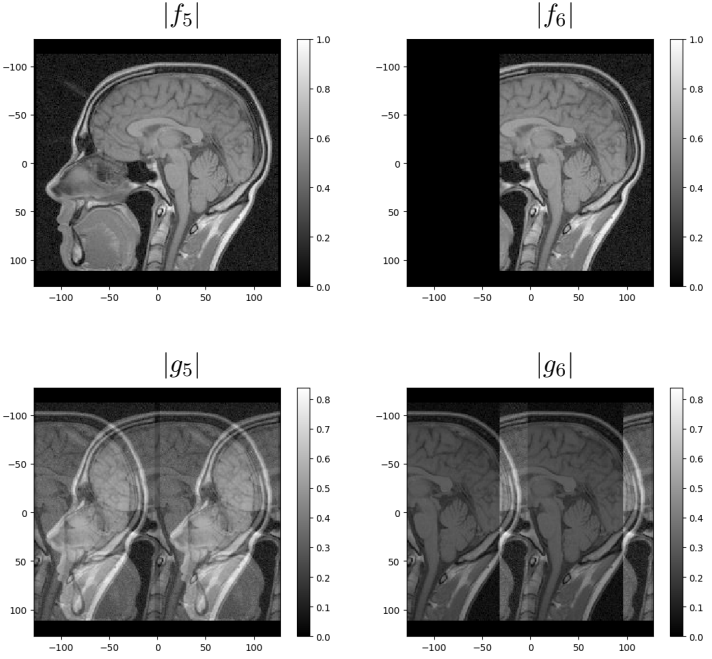

Notice that $c_6$ weights contributions from pixels in the range $-32 \leq c < 0$ exactly the same as those in $96 \leq c < 128$.

Therefore the $c_6$ image contains no information that is useful for separating these two bands of pixels.

Similar statements apply for $c_5$.

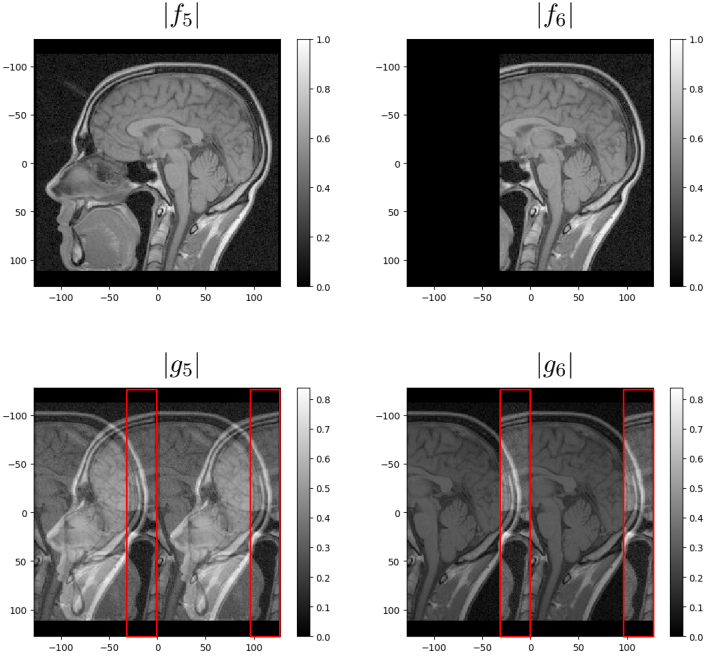
# Images From Coils 5 and 6

$g_5$, $g_6$ are after omitting odd numbered columns from $|f_5|$, $|f_6|$.



# Images From Coils 5 and 6

Highlighted regions are identical: both represent sum $f[r,c]+f[r,c+128]$.



## Conclusions

Magnetic Resonance Images are amazing – revealing deep tissue structure while being completely non-invasive.

Magnetic Resonance Images are acquired by sampling the Fourier representation of the proton density function.

Improving the imaging speed is an area of active research.

Naive methods (such as undersampling) that work in convention spatial imaging modalities are not applicable in MRI.

Magnetic Resonance Imaging can be made faster using multiple readout coils, which enables parallel acquisition of undersampled k-space data.