6.003: Signal Processing

Communications Systems

- Matching Signals to Communications Media
- Amplitude Modulation
- Frequency-Division Multiplexing
- Radio Receivers

- Quiz 2 grades are posted
- HW 9 is posted
  - just three problems
  - no lab, no check-in

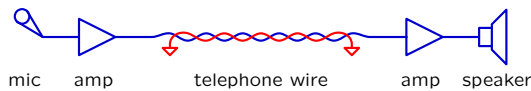*November 4, 2021*

Communications Systems

Beginning with commercial radio (1900s), communications technologies continue to be among the fastest growing applications of signal processing.

Examples:
- cellular communications
- wifi
- broadband
- bluetooth
- GPS (Global Positioning System)
- IOT (Internet of Things)
  - smart house / smart appliances
  - smart car
  - medical devices
- cable
- private networks: fire departments, police
- radar and navigation systems

Telephone

Popular thirst for communications has been evident since the early days of telephony.

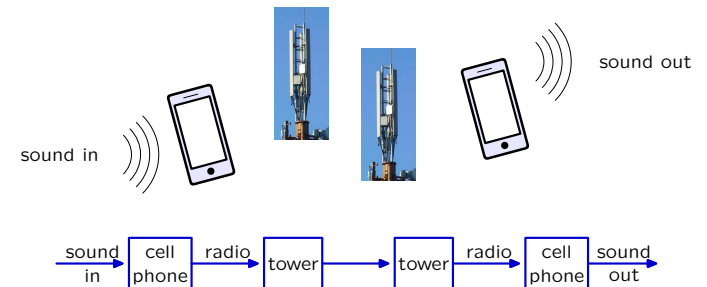mic   amp   telephone wire   amp   speaker

Patented by Alexander Graham Bell (1876), this technology flourished first as a network of **copper wires** and later as **optical fibers** ("long-distance" network) connecting virtually every household in the US by the 1980s.

Bell Labs became a premier research facility, developing information theory and a host of wired and wireless communications technologies that built on that theory, as well as hardware innovations such as the transistor and the laser.

Cellular Communication

First demonstrated by Motorola (1973), cellular communications quickly revolutionized the field. There are now more cell phones than people in the world.
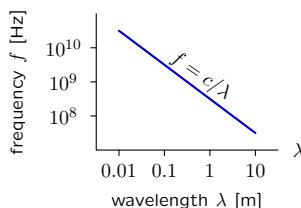

sound in   sound out

| sound in | cell phone | radio | tower | | tower | radio | cell phone | sound out |

Much of the popularity and convenience of cellular communications is that the communication is **wireless** (at least to the local tower).

Wireless Communication

Wireless signals are transmitted via electromagnetic (E/M) waves.

mic   amp   E/M wave   amp   speaker

For energy-efficient transmission and reception, the dimensions of the antenna should be on the order of the wavelength. But the wavelength of an electromagnetic wave depends on frequency.

Furthermore, the important frequencies depend on the application.

Matching Signals to Communications Media

A key problem in the design of any communications system is matching characteristics of the signal to those of the media.

Telephone-quality speech contains frequencies from 200 Hz to 3000 Hz.

How large must the antenna be for efficient transmission and reception of E/M waves?

Matching Signals to Communications Media

What frequency E/M wave is well matched to an antenna with a length of 10 cm (about 4 inches)?

Matching Signals to Communications Media

Different matching schemes are used to support different signals (from voice to streaming video) communicating through different media (E/M, optical fibers, coaxial cables).

Today we will introduce simple matching strategies based on **modulation**, which underlie virtually all matching schemes.

Matching Signals to Communications Media

How can we use frequencies in one band to communicate messages in a different band?

One simple method is based on the frequency-shift property of Fourier transforms.

If
$$x(t) \quad \overset{\text{CTFT}}{\iff} \quad X(\omega)$$

then
$$y(t) = e^{j\omega_c t} x(t) \quad \overset{\text{CTFT}}{\iff} \quad Y(\omega) = X(\omega - \omega_c)$$

$$\overline{Y(\omega) = \int \left(e^{j\omega_c t} x(t) \right) e^{-j\omega t} \, dt = \int x(t) e^{-j(\omega - \omega_c)t} \, dt}$$

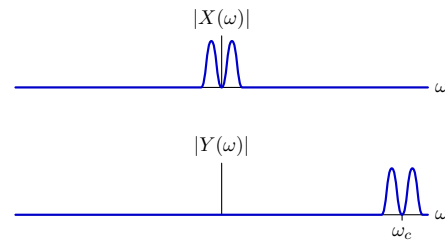Let $\lambda = \omega - \omega_c$.
$$Y(\lambda + \omega_c) = \int x(t) e^{-j\lambda t} \, dt = X(\lambda)$$

Let $\omega = \lambda + \omega_c$.
$$Y(\omega) = X(\omega - \omega_c)$$

Matching Signals to Communications Media

Multiplying a signal $x(t)$ by $e^{j\omega_c t}$ shifts the frequency content of $x(t)$ upward in frequency by ω_c.

The problem with this idea is that the signal $e^{j\omega_c t} x(t)$ has imaginary parts even when $x(t)$ is purely real.

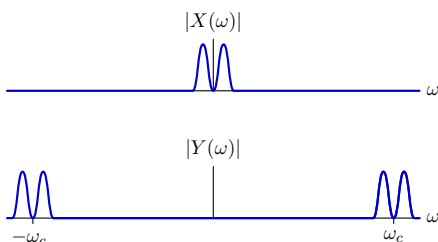## Matching Signals to Communications Media

A purely real-valued signal results if we shift a copy of $X(\omega)$ upward in frequency by ω_c and a second copy downward by the same amount.

If
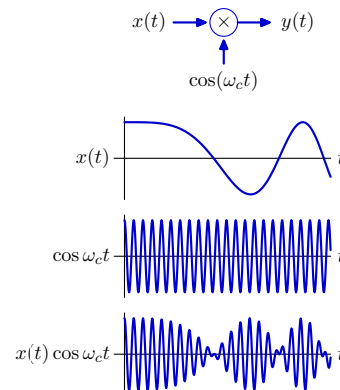$$x(t) \quad \overset{\text{CTFT}}{\iff} \quad X(\omega)$$

then
$$e^{j\omega_c t} x(t) + e^{-j\omega_c t} x(t) = 2\cos(\omega_c t) x(t) \quad \overset{\text{CTFT}}{\iff} \quad X(\omega - \omega_c) + X(\omega + \omega_c)$$

We refer to this scheme as **amplitude modulation.**

Amplitude Modulation (Time Domain)

Multiplying a signal by a sinusoidal "carrier" is called amplitude modulation. The signal "modulates" the amplitude of the carrier.

Multiplication Property of Fourier Transform

Multiplication in time corresponds to convolution in frequency.

Let

$$z(t) = x(t)y(t)$$

then

$$Z(\omega) = \int x(t)y(t)e^{-j\omega t}\, dt$$

Substitute $y(t) = \dfrac{1}{2\pi}\int Y(\omega)e^{j\omega t}d\omega = \dfrac{1}{2\pi}\int Y(\lambda)e^{j\lambda t}d\lambda$.
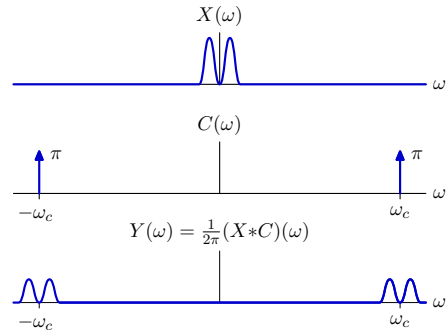
$$Z(\omega) = \int x(t)\Big(\frac{1}{2\pi}\int Y(\lambda)e^{j\lambda t}\, d\lambda\Big)e^{-j\omega t}\, dt$$

$$= \frac{1}{2\pi}\int Y(\lambda)\Big(\int x(t)e^{-j(\omega-\lambda)t}dt\Big)\, d\lambda$$

$$= \frac{1}{2\pi}\int Y(\lambda)X(\omega-\lambda)\, d\lambda = \frac{1}{2\pi}(X*Y)(\omega)$$

This result is the **dual** of filtering, where convolution in time corresponds to multiplication in frequency.

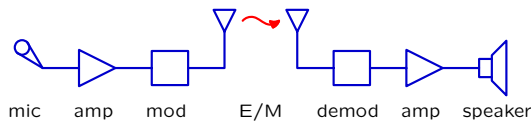## Amplitude Modulation (Frequency Domain)

Multiplication in time corresponds to convolution in frequency.

Let $X(\omega)$ represent the Fourier transform of the signal to be transmitted. Let $C(\omega)$ represent the Fourier transform of $\cos(\omega_c t) = \frac{1}{2}\big(e^{j\omega_c t} + e^{-j\omega_c t}\big)$. Then $Y(\omega)$ is the result of convolving $X(\omega)$ with $C(\omega)$.

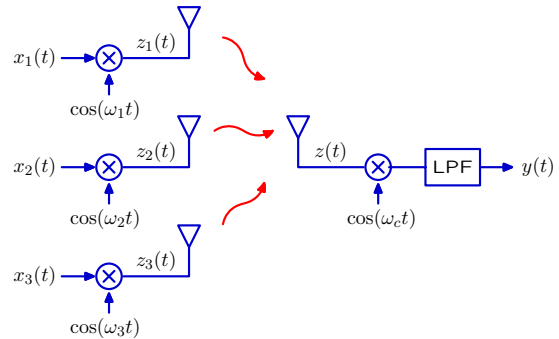## Demodulating the Received Signal

We can match the signal to the medium with a modulator as shown by the **mod** box below. But then we must demodulate the received signal to get back our original message (the **demod** box below).

mic   amp   mod   E/M   demod   amp   speaker

How can we recover the original message from the modulated signal?

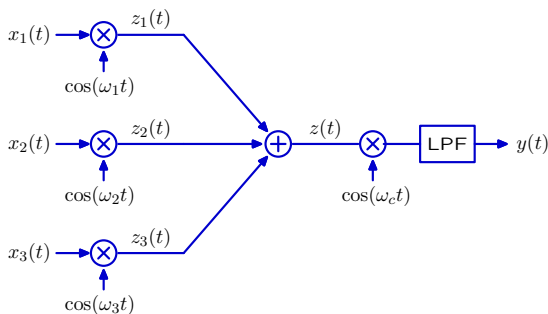## Frequency-Division Multiplexing

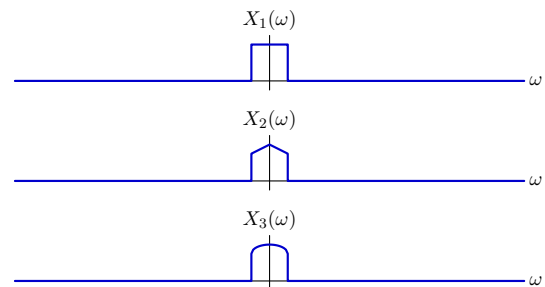Multiple transmitters can co-exist, as long as the frequencies that they transmit do not overlap.

Frequency-Division Multiplexing
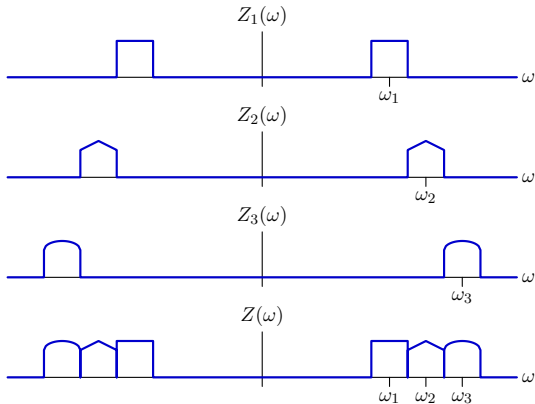
To first order, multiple transmitters simply sum.

Frequency-Division Multiplexing

Multiple transmitters can co-exist, as long as the frequencies that they transmit do not overlap.

Frequency-Division Multiplexing

Multiple transmitters can co-exist, as long as the frequencies that they transmit do not overlap.

$Z_1(\omega)$

ω_1

ω

$Z_2(\omega)$

ω_2

ω

$Z_3(\omega)$

ω_3

ω

$Z(\omega)$

ω_1 ω_2 ω_3

ω

Broadcast Radio

"Broadcast" radio was championed by David Sarnoff, who previously worked at Marconi Wireless Telegraphy Company (point-to-point).
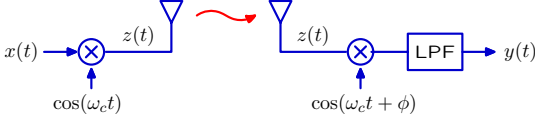
- envisioned "radio music boxes"
- analogous to newspaper, but at speed of light
- receiver must be cheap (as with newsprint)
- transmitter can be expensive (as with printing press)

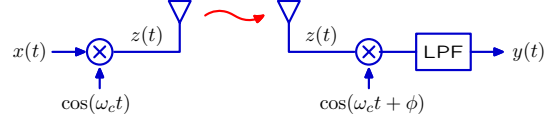Sarnoff (left) and Marconi (right)

Inexpensive Radio Receiver

An inexpensive receiver was needed to make broadcast radio successful. Synchronous demodulation is inexpensive, but requires knowing the carrier signal exactly.

$x(t)$ $\otimes$ $z(t)$

$\cos(\omega_c t)$

$z(t)$ $\otimes$ LPF $y(t)$

$\cos(\omega_c t + \phi)$

What happens if there is a phase shift ϕ between the signal used to modulate and the one used to demodulate?

Inexpensive Radio Receiver

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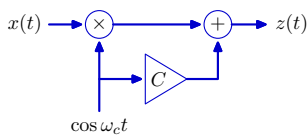
$x(t)$ $\otimes$ $z(t)$

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$\cos(\omega_c t + \phi)$

What happens if there is a phase shift ϕ between the signal used to modulate and the one used to demodulate?

$$y(t) = x(t)\cos(\omega_c t)\cos(\omega_c t + \phi)$$
$$= \frac{1}{2}x(t)\Big(\cos\phi + \cos(2\omega_c t + \phi)\Big)$$
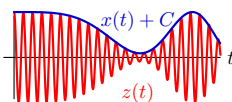$$= \frac{1}{2}x(t)\cos\phi + \frac{1}{2}x(t)\cos(2\omega_c t + \phi)$$

The second term is at a high frequency, so we can filter it out.
But multiplying by $\cos\phi$ in the first term is a problem: the signal "fades."
For example, if $\phi = \frac{\pi}{2}$, there is no output at all!

AM with Carrier

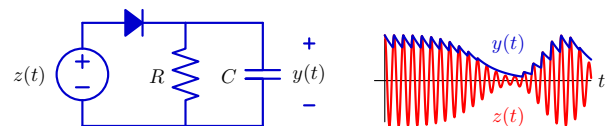One way to synchronize the sender and receiver is to send the carrier along with the message.

$x(t)$ $\otimes$ $+$ $z(t)$

C

$\cos \omega_c t$

$$z(t) = x(t)\cos\omega_c t + C\cos\omega_c t = (x(t) + C)\cos\omega_c t$$

$x(t) + C$

t

$z(t)$

Adding carrier is equivalent to shifting the DC value of $x(t)$.
If we shift the DC value sufficiently, the message is easy to decode: it is just the envelope (minus the DC shift).

Inexpensive Radio Receiver

If the carrier frequency is much greater than the highest frequency in the message, AM with carrier can be demodulated with a peak detector.

$z(t)$ R C $+$ $y(t)$ $-$
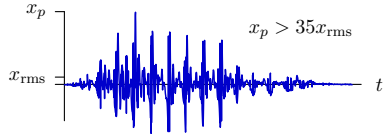
$y(t)$

t

$z(t)$

In AM radio, the highest frequency in the message is 5 kHz and the carrier frequency is between 500 kHz and 1500 kHz.

This circuit is simple and inexpensive.

But there is a problem.

Inexpensive Radio Receiver

AM with carrier requires more power to transmit the carrier than to transmit the message!
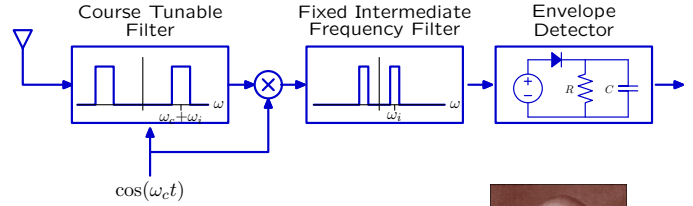

$$x_p > 35 x_{\text{rms}}$$

Speech sounds have high crest factors (peak value divided by rms value). Envelope detection will only work if the DC offset C is larger than x_p.

The power needed to transmit the carrier can be $35^2 \approx 1000\times$ that needed to transmit the message.

Okay for broadcast radio (WBZ: 50 kwatts).
Not for point-to-point (cell phone batteries wouldn't last long!).

Superheterodyne Receiver

Edwin Howard Armstrong invented the superheterodyne receiver, which made broadcast AM practical.
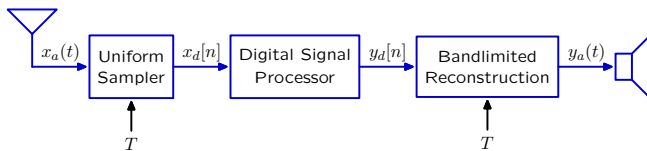

$$\cos(\omega_c t)$$

Edwin Howard Armstrong also invented and patented the "regenerative" (positive feedback) circuit for amplifying radio signals (while he was a junior at Columbia University). He also invented wideband FM.

Digital Radio

Today's radios are very different from those that launched broadcast radio.

Some issues remain the same:
- power utilization
- bandwidth limitations

Other issues are newer:
- more users
- more messages per user
- more different kinds of messages (audio, video, data)
- privacy and security

Signal processing plays an important role in all of these areas.

Summary

A key problem in communications is matching the signal to the medium.

Frequency modulation is an effective way to use frequencies in one band to communication messages in a different band.

Frequency modulation is based on the multiplication property of the Fourier transform.
- Multiplication in time corresponds to convolution in frequency.

The frequency shift property of the Fourier transform is a special case of the multiplication property.
- Multiplication by a sinusoid shifts the frequency content of a signal.