6.003 Quiz 1 Fall 2021

Name:

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use one 8.5×11 sheet of paper (two sides).

You may NOT use any electronic devices (such as calculators and phones).

If you have questions, please **come to us at the front** to ask them.

Please enter all solutions in the boxes provided.

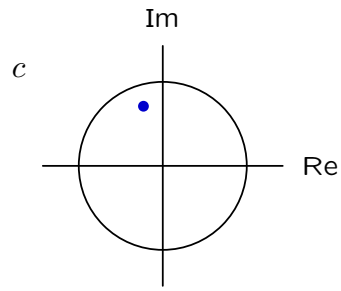
Work on other pages with QR codes will be considered for partial credit.
Please provide a note if you continue work on worksheets at the end of the exam (pages 13 and higher).

Trigonometric Identities Reference

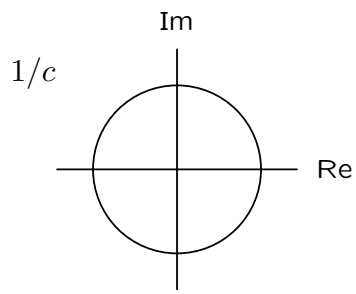
$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$$
$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$$
$$\cos(a) + \cos(b) = 2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\cos(a) - \cos(b) = -2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\sin(a) + \sin(b) = 2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)$$
$$\sin(a) - \sin(b) = 2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)$$
$$\cos(a+b)+\cos(a-b) = 2\cos(a)\cos(b)$$
$$\cos(a+b)-\cos(a-b) = -2\sin(a)\sin(b)$$
$$\sin(a+b)+\sin(a-b) = 2\sin(a)\cos(b)$$
$$\sin(a+b)-\sin(a-b) = 2\cos(a)\sin(b)$$
$$2\cos(a)\cos(b) = \cos(a-b)+\cos(a+b)$$
$$2\sin(a)\sin(b) = \cos(a-b)-\cos(a+b)$$
$$2\sin(a)\cos(b) = \sin(a+b)+\sin(a-b)$$
$$2\cos(a)\sin(b) = \sin(a+b)-\sin(a-b)$$

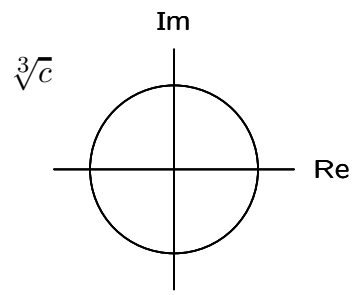
1 Complex Numbers (24 points)

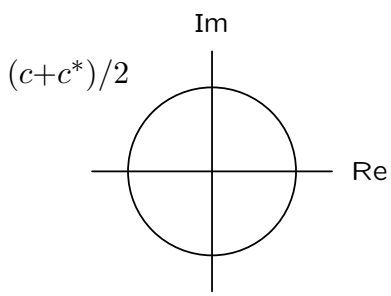
Let c represent the complex number shown by a filled dot in the following diagram, where the real and imaginary parts of c are shown on the horizontal and vertical axes, respectively, and the circle has a radius of 1.

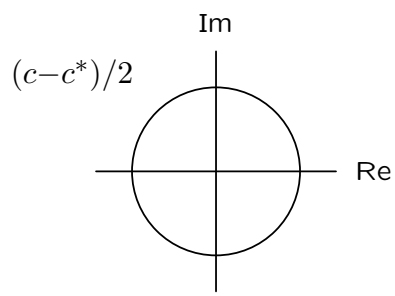


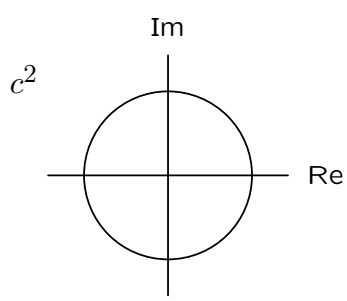
Below are eight complex-valued functions of c, each paired with a depiction of the complex plane demarked by the unit circle. Evaluate each expression and mark its value on the complex plane with a dot. If the expression can represent multiple complex numbers, mark at least two of them.

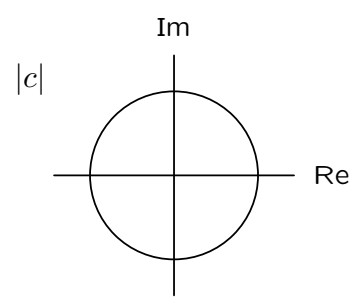


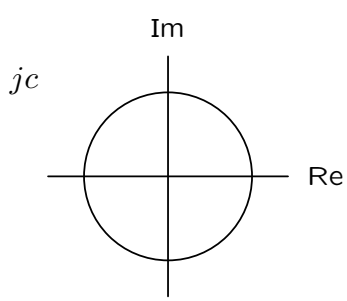


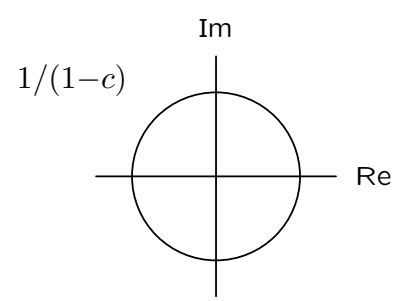












Worksheet (intentionally blank)

2 Sampling Sinuoids (25 points)

Let $f(t)$ represent the following continuous-time signal:

$$f(t) = 4\cos(300\pi t) + 2\sin(400\pi t) + \cos(600\pi t)$$

Part a. Let $f_1[n]$ represent a discrete-time signal that is obtained by samping $f(t)$ with sampling frequency $f_s = 100\,\text{Hz}$, so that

$$f_1[n] = f(n/f_s)$$

Determine the fundamental (shortest) period of $f_1[n]$. Your answer should be a number or a numeric expression that can include common constants (such as π).

Enter your answer in the box below. Enter **none** if $f_1[n]$ is not periodic.

fundamental period: [ ]

Part b. Let $f_2[n]$ represent a discrete-time signal that is obtained by samping $f(t)$ with sampling frequency $f_s = 200\,\text{Hz}$, so that

$$f_2[n] = f(n/f_s)$$

Determine the fundamental (shortest) period of $f_2[n]$. Your answer should be a number or a numeric expression that can include common constants (such as π).

Enter your answer in the box below. Enter **none** if $f_2[n]$ is not periodic.

fundamental period: [ ]

Part c. Let $f_3[n]$ represent a discrete-time signal that is obtained by samping $f(t)$ with sampling frequency $f_s = 300\,\text{Hz}$, so that

$$f_3[n] = f(n/f_s)$$

Determine the fundamental (shortest) period of $f_3[n]$. Your answer should be a number or a numeric expression that can include common constants (such as π).

Enter your answer in the box below. Enter **none** if $f_3[n]$ is not periodic.

fundamental period: [ ]

Part d. Let $f_4[n]$ represent a discrete-time signal that is obtained by samping $f(t)$ with sampling frequency $f_s = 1200\,\text{Hz}$, so that

$$f_4[n] = f(n/f_s)$$

Determine the fundamental (shortest) period of $f_4[n]$. Your answer should be a number or a numeric expression that can include common constants (such as π).

Enter your answer in the box below. Enter **none** if $f_4[n]$ is not periodic.

fundamental period: [ ]

Part e. Let $f_5[n]$ represent a discrete-time signal that is obtained by samping $f(t)$ with sampling frequency $f_s =$ 2400 Hz, so that

$$f_5[n] = f(n/f_s)$$

Determine the fundamental (shortest) period of $f_5[n]$. Your answer should be a number or a numeric expression that can include common constants (such as π).

Enter your answer in the box below. Enter **none** if $f_5[n]$ is not periodic.

fundamental period: $\boxed{}$

3 Find All (24 points)

Consider the following signals:

$$f_1[n] = \frac{1}{2} + 6\cos(\pi n/2) + 4\cos(\pi n/5 - \pi/2)$$

$$f_2[n] = \cos(1.8\pi n) + 2\sin(2.7\pi n)$$

$$f_3[n] = \left|\sin(\pi n/10)\right| \qquad \text{; where } |x| \text{ represents the magnitude of } x$$

$$f_4[n] = \mathrm{Im}\left\{e^{j\left(2\pi n/20+\pi/2\right)}\right\} \qquad \text{; where } \mathrm{Im}\{x\} \text{ represents the imaginary part } x$$

Part a. Determine which (if any) of signals $f_1[n]$ through $f_4[n]$ are **symmetric** about n=0, and circle **all** of those signals below. Circle **None** if no signal is symmetric about n=0.

$f_1[n]$ $f_2[n]$ $f_3[n]$ $f_4[n]$ **None**

Part b. Determine which (if any) of signals $f_1[n]$ through $f_4[n]$ are **periodic** with a fundamental (smallest) period N=20, and circle **all** of those signals below. Circle **None** if no signal is periodic with fundamental period N=20.

$f_1[n]$ $f_2[n]$ $f_3[n]$ $f_4[n]$ **None**

Part c. Determine which (if any) of signals $f_1[n]$ through $f_4[n]$ can be represented by Fourier series with **purely imaginary** coefficients, and circle **all** of those entries below. Circle **None** if none of these signals can be represented by Fourier series with purely imaginary coefficients.

$f_1[n]$ $f_2[n]$ $f_3[n]$ $f_4[n]$ **None**

Part d. Determine which (if any) of signals $f_i[n]$ can be represented by Fourier series coefficients $F_i[k]$ that are symmetric functions of k (i.e., $F_i[k] = F_i[-k]$) and circle **all** of those entries below. Circle **None** if none of the Fourier series coefficients are symmetric functions of k.

$f_1[n]$ $f_2[n]$ $f_3[n]$ $f_4[n]$ **None**

4 Alternative Representations (27 points)

Let $f_1(t)$ represent a function of continuous time t that is represented by a trigonometric Fourier series:

$$f_1(t) = \sum_{k=1}^{\infty} \frac{\sin(kt)}{k}$$

Part a. What is the average value of $f_1(t)$? Your answer should be a number or numeric expression that can include common constants (such as π). Your answer should not include $f_1(t)$ or any integrals or infinite sums.

Enter your answer here:

Part b. Determine the numerical value of the following integral:

$$\int_0^{2\pi} f_1(t)\cos(3t)dt$$

Your answer should be a number or numeric expression that can include common constants (such as π). Your answer should not include $f_1(t)$ or any integrals or infinite sums.

Enter your answer here:

Part c. Determine the numerical value of the following integral:

$$\int_0^{2\pi} f_1(t)\sin(5t)dt$$

Your answer should be a number or numeric expression that can include common constants (such as π). Your answer should not include $f_1(t)$ or any integrals or infinite sums.

Enter your answer here:

Part d. The function $f_1(t)$ can also be expressed as a complex exponential Fourier series as follows:

$$f_1(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_o t}$$

Determine the numerical values of ω_o, a_{-2}, a_{-1}, a_0, a_1, a_2 and enter those values in the table below. Each answer should be a number or numeric expression that can include common constants (such as π). Your answers should not include $f_1(t)$ or any integrals or infinite sums.

ω_o:	
a_{-2}:	
a_{-1}:	
a_0:	
a_1:	
a_2:	

Part e. Let c_k and d_k represent the trigonometric Fourier series coefficients for a periodic function $f_2(t)$ of continuous time t with period T:

$$f_2(t) = f_2(t+T) = c_0 + \sum_{k=1}^{\infty} c_k \cos(2\pi kt/T) + \sum_{k=1}^{\infty} d_k \sin(2\pi kt/T)$$

Determine expressions for the trigonometric Fourier series coefficients c'_k and d'_k for $f_3(t) = \frac{d}{dt} f_2(t)$:

$$f_3(t) = \frac{d}{dt} f_2(t) = c'_0 + \sum_{k=1}^{\infty} c'_k \cos(2\pi kt/T) + \sum_{k=1}^{\infty} d'_k \sin(2\pi kt/T)$$

as functions of $\ldots c_0, c_1, c_2, \ldots$ and/or $\ldots d_0, d_1, d_2, \ldots$.

Enter expressions for $c'_0, c'_1, d'_1, c'_2, d'_2, c'_3$, and d'_3 in the table below. Your expressions may contain any combination of the original (unprimed) coefficients ($\ldots c_0, c_1, c_2, \ldots$ and $\ldots d_0, d_1, d_2, \ldots$) but must not include $f_2(t)$ or $f_3(t)$ or any integrals or infinite sums.

Notice that d'_0 is not defined, and it's box is x'd out of the table.

	c'_k	d'_k
0:		
1:		
2:		
3:		

Part f. Let a_k represent the complex exponential Fourier series coefficients for a periodic function $f_4(t)$ of continuous time t with period T:

$$f_4(t) = f_4(t+T) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$$

Determine expressions for the complex exponential Fourier series coefficients a'_k for $f_5(t) = f_4(t)\cos(2\pi t/T)$

$$f_5(t) = f_4(t)\cos(2\pi t/T) = \sum_{k=-\infty}^{\infty} a'_k e^{j2\pi kt/T}$$

as functions of the original (unprimed) coefficients (a_k).

Enter expressions for a'_{-2}, a'_{-1}, a'_0, a'_1, and a'_2 in the table below. Your expressions may contain any combination of the original (unprimed) coefficients a_k but must not include $f_4(t)$ or $f_5(t)$ or any integrals or infinite sums.

a'_{-2}:	
a'_{-1}:	
a'_0:	
a'_1:	
a'_2:	

Worksheet (intentionally blank)

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