Name:

Kerberos (Athena) username:

Please WAIT until we tell you to begin.

This quiz is closed book, but you may use one 8.5 × 11 sheet of paper (two sides).

You may NOT use any electronic devices (such as calculators and phones).

If you have questions, please come to us at the front to ask them.

Please enter all solutions in the boxes provided.

Work on other pages with QR codes will be considered for partial credit.

Please provide a note if you continue work on worksheets at the end of the exam (pages 13 and higher).

Trigonometric Identities Reference

\[
\begin{align*}
\cos(a+b) &= \cos(a) \cos(b) - \sin(a) \sin(b) & \cos(a-b) &= \cos(a) \cos(b) + \sin(a) \sin(b) \\
\sin(a+b) &= \sin(a) \cos(b) + \cos(a) \sin(b) & \sin(a-b) &= \sin(a) \cos(b) - \cos(a) \sin(b) \\
\cos(a) + \cos(b) &= 2 \cos\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) & \cos(a) - \cos(b) &= -2 \sin\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \\
\sin(a) + \sin(b) &= 2 \sin\left(\frac{a+b}{2}\right) \cos\left(\frac{a-b}{2}\right) & \sin(a) - \sin(b) &= 2 \cos\left(\frac{a+b}{2}\right) \sin\left(\frac{a-b}{2}\right) \\
\cos(a+b)+\cos(a-b) &= 2 \cos(a) \cos(b) & \cos(a+b) - \cos(a-b) &= -2 \sin(a) \sin(b) \\
\sin(a+b)+\sin(a-b) &= 2 \sin(a) \cos(b) & \sin(a+b) - \sin(a-b) &= 2 \cos(a) \sin(b) \\
2\cos(a)\cos(b) &= \cos(a-b)+\cos(a+b) & 2\sin(a)\sin(b) &= \cos(a-b)-\cos(a+b) \\
2\sin(a)\cos(b) &= \sin(a+b)+\sin(a-b) & 2\cos(a)\sin(b) &= \sin(a+b)-\sin(a-b)
\end{align*}
\]
1 Complex Numbers (24 points)

Let \( c \) represent the complex number shown by a filled dot in the following diagram, where the real and imaginary parts of \( c \) are shown on the horizontal and vertical axes, respectively, and the circle has a radius of 1.

Below are eight complex-valued functions of \( c \), each paired with a depiction of the complex plane demarked by the unit circle. Evaluate each expression and mark its value on the complex plane with a dot. If the expression can represent multiple complex numbers, mark at least two of them.
Worksheet (intentionally blank)
2 **Sampling Sinuoids (25 points)**

Let \( f(t) \) represent the following continuous-time signal:
\[
f(t) = 4 \cos(300\pi t) + 2 \sin(400\pi t) + \cos(600\pi t)
\]

**Part a.** Let \( f_1[n] \) represent a discrete-time signal that is obtained by sampling \( f(t) \) with sampling frequency \( f_s = 100 \) Hz, so that
\[
f_1[n] = f(n/f_s)
\]
Determine the fundamental (shortest) period of \( f_1[n] \). Your answer should be a number or a numeric expression that can include common constants (such as \( \pi \)).

Enter your answer in the box below. Enter **none** if \( f_1[n] \) is not periodic.

fundamental period: 

**Part b.** Let \( f_2[n] \) represent a discrete-time signal that is obtained by sampling \( f(t) \) with sampling frequency \( f_s = 200 \) Hz, so that
\[
f_2[n] = f(n/f_s)
\]
Determine the fundamental (shortest) period of \( f_2[n] \). Your answer should be a number or a numeric expression that can include common constants (such as \( \pi \)).

Enter your answer in the box below. Enter **none** if \( f_2[n] \) is not periodic.

fundamental period: 

**Part c.** Let \( f_3[n] \) represent a discrete-time signal that is obtained by sampling \( f(t) \) with sampling frequency \( f_s = 300 \) Hz, so that
\[
f_3[n] = f(n/f_s)
\]
Determine the fundamental (shortest) period of \( f_3[n] \). Your answer should be a number or a numeric expression that can include common constants (such as \( \pi \)).
Enter your answer in the box below. Enter **none** if \( f_3[n] \) is not periodic.

fundamental period: ____________

**Part d.** Let \( f_4[n] \) represent a discrete-time signal that is obtained by sampling \( f(t) \) with sampling frequency \( f_s = 1200 \) Hz, so that
\[
f_4[n] = f(n/f_s)
\]
Determine the fundamental (shortest) period of \( f_4[n] \). Your answer should be a number or a numeric expression that can include common constants (such as \( \pi \)).
Enter your answer in the box below. Enter **none** if \( f_4[n] \) is not periodic.

fundamental period: ____________
Part e. Let $f_5[n]$ represent a discrete-time signal that is obtained by sampling $f(t)$ with sampling frequency $f_s = 2400$ Hz, so that

$$f_5[n] = f(n/f_s)$$

Determine the fundamental (shortest) period of $f_5[n]$. Your answer should be a number or a numeric expression that can include common constants (such as $\pi$).

Enter your answer in the box below. Enter **none** if $f_5[n]$ is not periodic.

fundamental period: none
3 Find All (24 points)

Consider the following signals:

\[ f_1[n] = \frac{1}{2} + 6 \cos(\pi n/2) + 4 \cos(\pi n/5 - \pi/2) \]
\[ f_2[n] = \cos(1.8\pi n) + 2 \sin(2.7\pi n) \]
\[ f_3[n] = \left| \sin(\pi n/10) \right| \quad \text{; where } |x| \text{ represents the magnitude of } x \]
\[ f_4[n] = \text{Im}\left\{ e^{i(2\pi n/20 + \pi/2)} \right\} \quad \text{; where } \text{Im}\{x\} \text{ represents the imaginary part of } x \]

**Part a.** Determine which (if any) of signals \( f_1[n] \) through \( f_4[n] \) are symmetric about \( n=0 \), and circle all of those signals below. Circle **None** if no signal is symmetric about \( n=0 \).

\[ f_1[n] \quad f_2[n] \quad f_3[n] \quad f_4[n] \quad \text{None} \]

**Part b.** Determine which (if any) of signals \( f_1[n] \) through \( f_4[n] \) are periodic with a fundamental (smallest) period \( N=20 \), and circle all of those signals below. Circle **None** if no signal is periodic with fundamental period \( N=20 \).

\[ f_1[n] \quad f_2[n] \quad f_3[n] \quad f_4[n] \quad \text{None} \]
Part c. Determine which (if any) of signals $f_1[n]$ through $f_4[n]$ can be represented by Fourier series with purely imaginary coefficients, and circle all of those entries below. Circle None if none of these signals can be represented by Fourier series with purely imaginary coefficients.

\[
\begin{array}{cccc}
  f_1[n] & f_2[n] & f_3[n] & f_4[n] \\
  \text{None} & & & \\
\end{array}
\]

Part d. Determine which (if any) of signals $f_i[n]$ can be represented by Fourier series coefficients $F_i[k]$ that are symmetric functions of $k$ (i.e., $F_i[k] = F_i[-k]$) and circle all of those entries below. Circle None if none of the Fourier series coefficients are symmetric functions of $k$.

\[
\begin{array}{cccc}
  f_1[n] & f_2[n] & f_3[n] & f_4[n] \\
  \text{None} & & & \\
\end{array}
\]
4 Alternative Representations (27 points)

Let \( f_1(t) \) represent a function of continuous time \( t \) that is represented by a trigonometric Fourier series:

\[
f_1(t) = \sum_{k=1}^{\infty} \frac{\sin(kt)}{k}
\]

**Part a.** What is the average value of \( f_1(t) \)? Your answer should be a number or numeric expression that can include common constants (such as \( \pi \)). Your answer should not include \( f_1(t) \) or any integrals or infinite sums.

Enter your answer here: 

**Part b.** Determine the numerical value of the following integral:

\[
\int_{0}^{2\pi} f_1(t) \cos(3t) \, dt
\]

Your answer should be a number or numeric expression that can include common constants (such as \( \pi \)). Your answer should not include \( f_1(t) \) or any integrals or infinite sums.

Enter your answer here: 

Part c. Determine the numerical value of the following integral:

\[
\int_{0}^{2\pi} f_1(t) \sin(5t) \, dt
\]

Your answer should be a number or numeric expression that can include common constants (such as \(\pi\)). Your answer should not include \(f_1(t)\) or any integrals or infinite sums.

Enter your answer here: 

Part d. The function \(f_1(t)\) can also be expressed as a complex exponential Fourier series as follows:

\[
f_1(t) = \sum_{k=-\infty}^{\infty} a_k e^{j k \omega_o t}
\]

Determine the numerical values of \(\omega_o\), \(a_{-2}\), \(a_{-1}\), \(a_0\), \(a_1\), \(a_2\) and enter those values in the table below. Each answer should be a number or numeric expression that can include common constants (such as \(\pi\)). Your answers should not include \(f_1(t)\) or any integrals or infinite sums.
Part e. Let $c_k$ and $d_k$ represent the trigonometric Fourier series coefficients for a periodic function $f_2(t)$ of continuous time $t$ with period $T$:

$$f_2(t) = f_2(t + T) = c_0 + \sum_{k=1}^{\infty} c_k \cos\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{\infty} d_k \sin\left(\frac{2\pi kt}{T}\right)$$

Determine expressions for the trigonometric Fourier series coefficients $c'_k$ and $d'_k$ for $f_3(t) = \frac{d}{dt}f_2(t)$:

$$f_3(t) = \frac{d}{dt}f_2(t) = c'_0 + \sum_{k=1}^{\infty} c'_k \cos\left(\frac{2\pi kt}{T}\right) + \sum_{k=1}^{\infty} d'_k \sin\left(\frac{2\pi kt}{T}\right)$$

as functions of $\ldots c_0, c_1, c_2, \ldots$ and/or $\ldots d_0, d_1, d_2, \ldots$.

Enter expressions for $c'_0, c'_1, d'_1, c'_2, d'_2, c'_3$, and $d'_3$ in the table below. Your expressions may contain any combination of the original (unprimed) coefficients $\ldots c_0, c_1, c_2, \ldots$ and $\ldots d_0, d_1, d_2, \ldots$ but must not include $f_2(t)$ or $f_3(t)$ or any integrals or infinite sums.

Notice that $d'_0$ is not defined, and it’s box is x’d out of the table.
Part f. Let $a_k$ represent the complex exponential Fourier series coefficients for a periodic function $f_4(t)$ of continuous time $t$ with period $T$:

$$f_4(t) = f_4(t + T) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$$

Determine expressions for the complex exponential Fourier series coefficients $a'_k$ for $f_5(t) = f_4(t) \cos(2\pi t/T)$

$$f_5(t) = f_4(t) \cos(2\pi t/T) = \sum_{k=-\infty}^{\infty} a'_k e^{j2\pi kt/T}$$

as functions of the original (unprimed) coefficients ($a_k$).

Enter expressions for $a'_{-2}$, $a'_{-1}$, $a'_0$, $a'_1$, and $a'_2$ in the table below. Your expressions may contain any combination of the original (unprimed) coefficients $a_k$ but must not include $f_4(t)$ or $f_5(t)$ or any integrals or infinite sums.

| $a'_{-2}$: | 
| $a'_{-1}$: | 
| $a'_0$: | 
| $a'_1$: | 
| $a'_2$: |
Worksheet (intentionally blank)
Worksheet (intentionally blank)
Worksheet (intentionally blank)