6.003: Signal Processing

Discrete Fourier Transform (DFT)

Definition and comparison to other Fourier representations.

	analysis	synthesis
DFT:	$X[k]=\frac{1}{N}\sum_{n=0}^{N-1}x[n]e^{-j\frac{2\pi}{N}kn}$	$x[n]=\sum_{k=0}^{N-1}X[k]e^{j\frac{2\pi}{N}kn}$
DTFS:	$X[k]=\frac{1}{N}\sum_{n=\langle N\rangle}x[n]e^{-j\frac{2\pi}{N}kn}$	$x[n]=\sum_{k=\langle N\rangle}X[k]e^{j\frac{2\pi}{N}kn}$
DTFT:	$X(\Omega)=\sum_{n=-\infty}^{\infty}x[n]e^{-j\Omega n}$	$x[n]=\frac{1}{2\pi}\int_{2\pi}X(\Omega)e^{j\Omega n}d\Omega$

29 September 2020

Analyzing Frequency Content of Arbitrary Signals

Why use a DFT?

Fourier Series: conceptually simple, but limited to periodic signals.

Fourier Transforms: arbitrary signals, but continuous domain (ω, Ω).

Discrete Fourier Transform: arbitrary DT signals (finite length)

- Discrete in both domains: nice for computation
- FFT: Efficient means of computation

Today: using the DFT to analyze frequency content of a signal.

Single Sinusoid

Create three signals of the following form:

$$x_1[n] = \cos(8\pi n/100)$$

$$x_2[n] = \cos(8\pi n/100 - \pi/4)$$

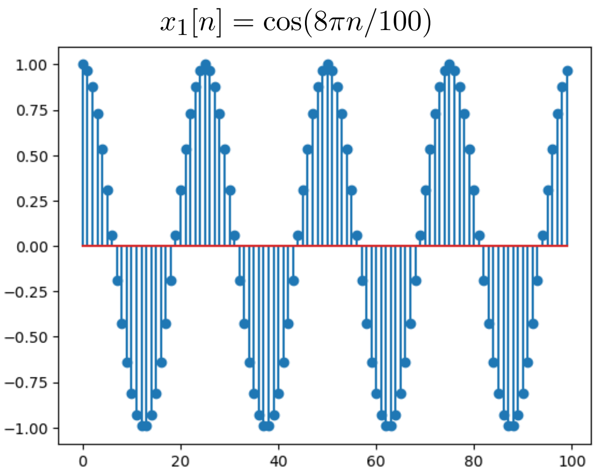
$$x_3[n] = \cos(9\pi n/100)$$

$$x_4[n] = \cos(9\pi n/100 - \pi/2)$$

Each should have a duration of 1 second and should use a sample frequency of 8kHz.

Compare the DFTs of the first 100 samples of each of these signals.

Single Sinusoid



What is the frequency of this tone if the sample rate is 8kHz?

Single Sinusoid

Write a program to calculate the DFT of an input sequence.

Use that program to calculate $X_1[k]$, which is the DFT of $x_1[n]$.

Single Sinusoid

Plot the magnitude of $X_1[\cdot]$.

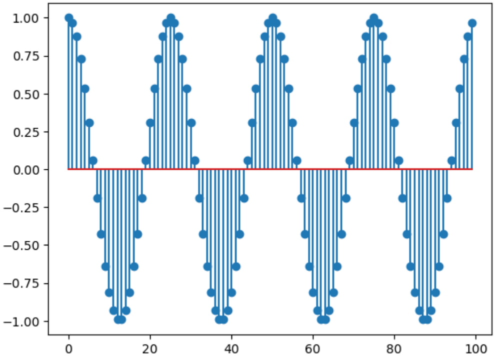
Frequency Scales

We can think of the DFT as having spectral resolution of $(2\pi/N)$ radians in DT, which is equivalent to (f_s/N) Hz in CT.

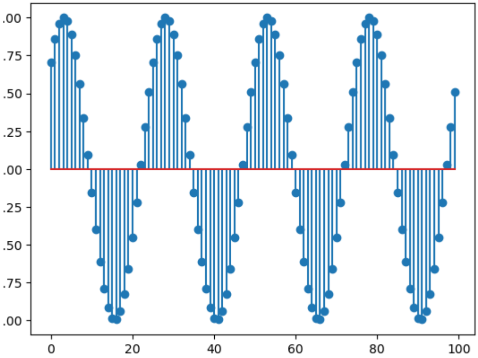
Compare Two Signals

How will plots of DFT magnitudes differ for the following signals?

$$x_1[n] = \cos(8\pi n/100)$$



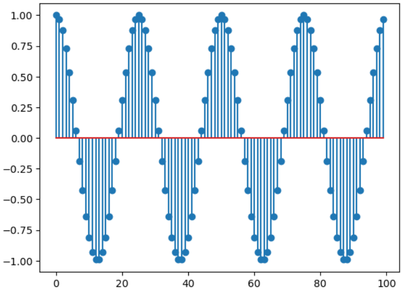
$$x_2[n] = \cos(8\pi n/100 - \pi/4)$$



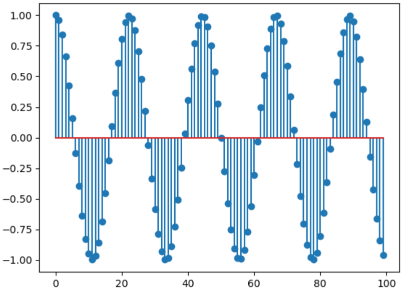
Compare Two Signals

How will plots of DFT magnitudes differ for the following signals?

$$x_1[n] = \cos(8\pi n/100)$$



$$x_3[n] = \cos(9\pi n/100)$$



Compare Two Signals

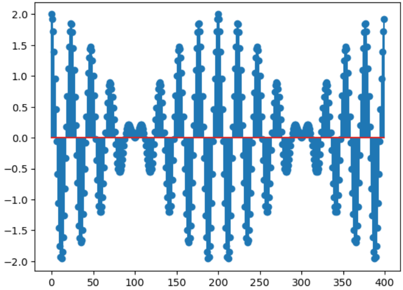
How will plots of DFT magnitudes differ for the following signals?

- $x_3[n] = \cos(9\pi n/100)$
- $x_4[n] = \cos(9\pi n/100 - \pi/2)$

Analyzing Signals with Multiple Frequencies

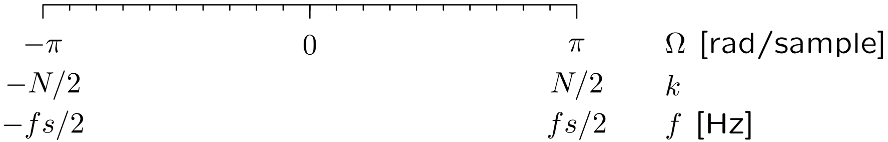
What is the minimum window size N needed to resolve $\Omega = 8\pi/100$ from $9\pi/100$?

$$x_5[n] = \cos(8\pi n/100) + \cos(9\pi n/100)$$



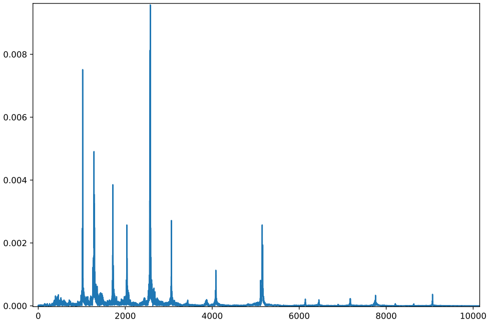
Frequency Scales

We can think of the DFT as having spectral resolution of $(2\pi/N)$ radians in DT, which is equivalent to (f_s/N) Hz in CT.



Check Yourself!

For a portion of the Chopin song containing only one chord, 250332 samples long, and recorded with a sampling rate of 48kHz, the DFT magnitudes look like:



Peaks in magnitude around $k \approx 1021, 1282, 1715, 2037, 2576, 3062, \ldots$

What are the frequencies (in Hz) of the notes being played?

What chord does this correspond to?