6.003: Signal Processing

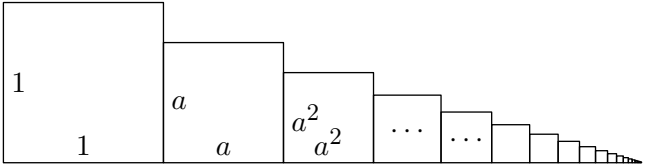
DTFT and Properties

- Math Preliminaries
- DTFT and Properties

24 September 2020

Squares

Squares that are $1{\times}1$, $a{\times}a$, $a^2{\times}a^2$, etc., are arranged side-by-side as shown below.



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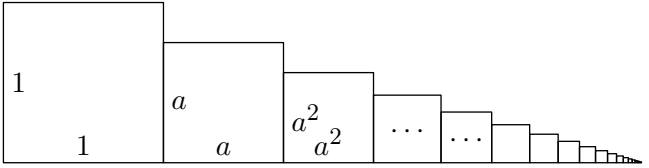

The upper left corners of these squares can be connected with a straight line.

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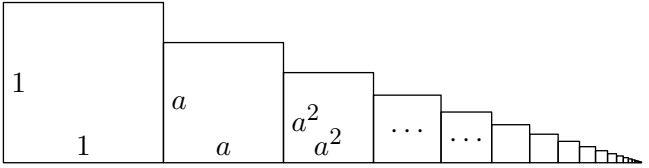

The upper left corners of these squares can be connected with a straight line. Determine the slope of the blue line.

Power Square

Find the sum of the numbers in the infinite quadrant shown below, where $a < 1$.

$$
\begin{array}{|cccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \\
a^4 & a^5 & a^6 & a^7 & a^8 & \cdots \\
a^3 & a^4 & a^5 & a^6 & a^7 & \cdots \\
a^2 & a^3 & a^4 & a^5 & a^6 & \cdots \\
a^1 & a^2 & a^3 & a^4 & a^5 & \cdots \\
a^0 & a^1 & a^2 & a^3 & a^4 & \cdots \\
\hline
\end{array}
$$

Discrete-Time Fourier Transform

Synthesis Equation

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega)\, e^{j\Omega n}\, d\Omega$$

Analysis Equation

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n}$$

Discrete-Time Fourier Transform

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Discrete-time Fourier Transform

Problem: Find the Fourier transform of the following signal.

$$x[n] = a^n u[n] \quad \text{where} \quad u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Sketch its magnitude and phase.

Inverse Discrete-Time Fourier Transform

Find the signal whose Fourier transform is

$$X(\Omega) = e^{-j3\Omega}$$

Discrete-Time Fourier Transform

Find the Fourier transforms of the following discrete-time signals.

- $x_1[n] = a^n u[n]$ where $u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases}$
- $x_2[n] = a^{(n-n_0)} u[n - n_0]$
- $x_3[n] = \mathrm{Sym}\{a^n u[n]\}$
- $x_4[n] = \mathrm{Asym}\{a^n u[n]\}$
- $x_5[n] = n a^n u[n]$

Discrete-Time Fourier Transform

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Discrete-Time Fourier Transform

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Discrete-Time Fourier Transform

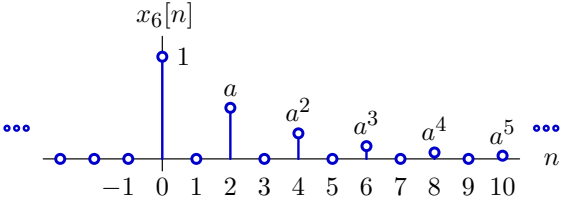
Find the Fourier transform of

$x_5[n] = na^n u[n]$

Discrete-Time Fourier Transform

Find the Fourier transform of $x_6[n]$:

$$x_6[n] = \begin{cases} (a)^{n/2} & n = 0, 2, 4, 6, 8, ..., \infty \\ 0 & \text{otherwise} \end{cases}$$



Plot the magnitude and angle of $X_6(\Omega)$ versus Ω.

Discrete-Time Fourier Transform

$$x_6[n] = \begin{cases} (a)^{n/2} & n = 0, 2, 4, 6, 8, ..., \infty \\ 0 & \text{otherwise} \end{cases}$$