6.003: Signal Processing

DTFT and Properties

- Math Preliminaries
- DTFT and Properties

24 September 2020
Squares

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Squares that are \(1 \times 1\), \(a \times a\), \(a^2 \times a^2\), etc., are arranged side-by-side as shown below.

The upper left corners of these squares can be connected with a straight line. Determine the slope of the blue line.
Power Square

Find the sum of the numbers in the infinite quadrant shown below, where \( a < 1 \).

\[
\begin{array}{cccccc}
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
a^4 & a^5 & a^6 & a^7 & a^8 & \cdots \\
a^3 & a^4 & a^5 & a^6 & a^7 & \cdots \\
a^2 & a^3 & a^4 & a^5 & a^6 & \cdots \\
a^1 & a^2 & a^3 & a^4 & a^5 & \cdots \\
a^0 & a^1 & a^2 & a^3 & a^4 & \cdots \\
\end{array}
\]
Discrete-Time Fourier Transform

**Synthesis Equation**

\[ x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\Omega) e^{j\Omega n} d\Omega \]

**Analysis Equation**

\[ X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} \]
Discrete-Time Fourier Transform

**Synthesis Equation**

\[ x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega) e^{j\Omega n} d\Omega \]

**Analysis Equation**

\[ X(\Omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\Omega n} \]
**Problem:** Find the Fourier transform of the following signal.

\[ x[n] = a^n u[n] \quad \text{where} \quad u[n] = \begin{cases} 
1 & \text{if } n \geq 0 \\
0 & \text{otherwise}
\end{cases} \]

Sketch its magnitude and phase.
Inverse Discrete-Time Fourier Transform

Find the signal whose Fourier transform is

\[ X(\Omega) = e^{-j3\Omega} \]
Discrete-Time Fourier Transform

Find the Fourier transforms of the following discrete-time signals.

- \( x_1[n] = a^n u[n] \) where \( u[n] = \begin{cases} 1 & \text{if } n \geq 0 \\ 0 & \text{otherwise} \end{cases} \)

- \( x_2[n] = a^{(n-n_0)} u[n - n_0] \)

- \( x_3[n] = \text{Sym}\{a^n u[n]\} \)

- \( x_4[n] = \text{Asym}\{a^n u[n]\} \)

- \( x_5[n] = na^n u[n] \)
Find the Fourier transform of

\[ x_2[n] = a^{(n-n_0)} u[n - n_0] \]
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Discrete-Time Fourier Transform

Find the Fourier transform of

\[ x_5[n] = n a^n u[n] \]
Find the Fourier transform of $x_6[n]$:

$$x_6[n] = \begin{cases} 
(a)^{n/2} & n = 0, 2, 4, 6, 8, \ldots, \infty \\
0 & \text{otherwise}
\end{cases}$$

Plot the magnitude and angle of $X_6(\Omega)$ versus $\Omega$. 
Discrete-Time Fourier Transform

\[ x_6[n] = \begin{cases} 
   (a)^{n/2} & n = 0, 2, 4, 6, 8, \ldots, \infty \\
   0 & \text{otherwise} 
\end{cases} \]