6.003: Signal Processing

Short-term Fourier Transform

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What is 6.003?

What is a signal?

Abstractly, a **signal** is a function that conveys information.

**Signal processing** is about extracting meaningful information from signals, and/or manipulating information to produce new signals.

What is a transform?

Provide multiple views/perspectives on a signal.

Some information more clearly visible (and/or more easily manipulable) from one perspective than another.
Why Fourier?

Sinusoids are nice!

- prevalent in nature
- relevant to human perception
- mathematically convenient (particularly as complex exponentials)
**Time-varying Signals**

Real-world signals (i.e., speech, music, ...) often have frequency content that varies with time.

Problem with DFT: events that are local in time are global in frequency (and *vice versa*). Sudden changes and local variations can be difficult to detect.

Example: 2 tunes

How to tell them apart?
The short-time Fourier Transform is a compromise between time- and frequency-domain representations, representing the frequency content of the signal at various points in time.

Formally, we define the STFT of a signal $x$ as:

$$\text{STFT}\{x\}[m, k] = \sum_{n=0}^{N-1} x[n + m \times s]w[n]e^{-j \frac{2\pi k}{N} n}$$

where:

- $N$ is the length of a window
- $s$ is a “step size”
- $w[\cdot]$ is a window function
- $m$ is a time index, and $k$ is a frequency index
Short-time Fourier Transform

Conceptually, we are taking the DFT of successive windowed regions of the original signal (and these regions may overlap).
STFT and Spectrograms

The STFT enhances our ability to reason about the frequency content of signals at various points in time. It is often visualized using a spectrogram, which is defined to be the magnitude squared of the STFT.
Today

The Rest of Today: Examples of Spectrograms

Recitation: Windowing

PSet, Lab: Filtering, Spectrograms