Synthetic Aperture Optics
Subject Evaluations

Your feedback is important to us!

Please give feedback to the staff and future 6.003 students:  
http://registrar.mit.edu/subjectevaluation

Evaluations are open until Monday, December 17 at 9 am.

You will be able to view quantitative results at  
http://web.mit.edu/subjectevaluation/results.html
and student-written summaries at  
http://hkn.mit.edu/ug_sel.php
Final Exam

Monday, December 17, 1:30-4:30pm, Johnson Track (W34)

The exam is closed book. No electronic devices. You may use three 8.5x11” sheets of notes (front and back).

Coverage: all lectures, labs, recitations, and homeworks.

Optional Practice Exam

Thursday, December 13, 2-4pm, 1-190

Normal Office Hours

Thursday, December 13 and Sunday, December 16

Late Homework Submissions

Accepted until Monday, December 17, at 9am
Today’s Lecture and Lab

Fourier Transforms in Optical Imaging.

• Fourier Optics
• Synthetic Aperture Microscopy
• Synthetic Aperture Projection
Images from even the best microscopes are blurred.
Optical Imaging

A perfect lens transforms a spherical wave of light from the target into a spherical wave that converges to the image.

Blurring is inversely related to the diameter of the lens.
Optical Imaging

A perfect lens transforms a spherical wave of light from the target into a spherical wave that converges to the image.

Blurring is inversely related to the diameter of the lens.
Optical Imaging

A perfect lens transforms a spherical wave of light from the target into a spherical wave that converges to the image.

Blurring is inversely related to the diameter of the lens.
Optical Imaging

Blurring can be represented by convolving the image with the optical “point-spread-function” (impulse response).

Blurring is inversely related to the diameter of the lens.
Blurring can be represented by convolving the image with the optical "point-spread-function" (impulse response).

Blurring is inversely related to the diameter of the lens.
Optical Imaging

Blurring can be represented by convolving the image with the optical “point-spread-function” (impulse response).

Blurring is inversely related to the diameter of the lens.
Optical Imaging

Sharper imaging was the primary motivation of the enormous size (and associated cost) of the Hubble mirror (2.4 meter diameter).
Optical Imaging

Why does the size of the optic affect image resolution?

Why are small lenses and mirrors a problem?

A well-formed lens (or mirror) focuses light from points on a target to corresponding points in the image – regardless of lens size.

Fourier transforms provide insight to understand (and even overcome some of) these limitations.
Fourier Optics

If a target is located in the focal plane of a lens, light from a point on the target forms a plane wave as it passes through the lens.

If the target point lies on the axis of the lens, then the plane wave is perpendicular to the imaging plane.
Fourier Optics

If a target lies off the axis of the lens, then the plane wave is no longer perpendicular to the image plane.

There is a linearly increasing phase delay between the light in the plane wave and the image plane.

Furthermore, the phase delay is greater for points that are more distant from the axis of the lens.
Fourier Optics

Light from $x=0$ generates a plane wave, that is everywhere in phase at the imaging plane.

\[ \delta(x) \rightarrow 1 \]

Light from $x=x_o$ generates a plane wave with linearly increasing phase lag.

\[ \delta(x - x_o) \rightarrow e^{-j\omega x_o} \]
The target can be described as a collection of point sources of light.

\[
f(x) = \int f(x_o) \delta(x - x_o) \, dx_o
\]

The resulting image \( g(\omega x) \) is the superposition of plane waves, one for each point in the image.

\[
g(\omega x) = \int f(x_o) e^{-j\omega x x_o} \, dx_o = F(\omega x)
\]

and \( g(\omega x) \) is the Fourier transform of \( f(x) \).
Fourier optics: there is a Fourier relationship between a target and its projection by a focused lens.

What (if any) insight into image resolution can be obtained from Fourier optics?
Check Yourself

Fourier optics: there is a Fourier relationship between a target and its projection by a focused lens.

What (if any) insight into image resolution can be obtained from Fourier optics?

The highest spatial frequencies derive from the highest angle parts of the spherical wave from a target – parts that are lost with small lenses.
Microscopy with 6.003

Dennis M. Freeman  
Stanley S. Hong  
Jekwan Ryu  
Michael S. Mermelstein  
Berthold K. P. Horn
6.003 Model of a Microscope

Microscope = low-pass filter
Phase-Modulated Microscopy
Demonstration
Phase-Modulated Microscopy

Poster: \[ \cos(\omega_c y + f(x,y)) \]
Phase-Modulated Microscopy

Poster: \( \cos(\omega_c y + f(x,y)) \)

Projector: \( \cos(\omega_c y) \)
Phase-Modulated Microscopy

Poster:
\[ \cos(\omega_c y + f(x,y)) \]

Projector:
\[ \cos(\omega_c y) \]

Poster with Projector:
\[ \cos(\omega_c y) \cos(\omega_c y + f(x,y)) \]

Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies
Phase-Modulated Microscopy

$X(\omega)$

$-\omega_c \quad \omega_c$
visible

$-\omega_c \quad \omega_c$

$-2\omega_c \quad 2\omega_c$
low-pass

Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies
Phase-Modulated Microscopy

$X(\omega)$

$\omega$

$-\omega_c$ visible $\omega_c$

$\omega$

$-\omega_c$ $\omega_c$

low-pass

$2\omega_c$

Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies
Phase-Modulated Microscopy

Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies
Phase-Modulated Microscopy

Modulated illumination enables low-pass system (eyes) to detect high spatial frequencies
Optical transfer function

2 beams
Optical transfer function

3 beams
Optical transfer function

4 beams
Optical transfer function

5 beams
Optical transfer function

6 beams
Optical transfer function

7 beams
Standing-wave illumination spectrum

Thanks to M. Mermelstein
Standing-wave illumination spectrum

Thanks to M. Mermelstein
Standing-wave illumination spectrum

Thanks to M. Mermelstein
Standing-wave illumination spectrum

Thanks to M. Mermelstein
Standing-wave illumination spectrum

Thanks to M. Mermelstein
Standing-wave illumination spectrum

Thanks to M. Mermelstein
Standing-wave illumination spectrum

Thanks to M. Mermelstein
Standing-wave illumination spectrum

Thanks to M. Mermelstein
Standing-wave illumination spectrum

Thanks to M. Mermelstein
Standing-wave illumination spectrum

Thanks to M. Mermelstein
Experimental apparatus

Stanley S. Hong
Measurement of PSF

SWI apparatus

Isolated 200-nm-diameter fluorescent microsphere

Glass

Air

Objective lens

(Cross section, not to scale)
Measurement of PSF
Measurement of PSF
Measurement of PSF

Intensity (photons)

Distance (nm)
Measurement of PSF

![Graph showing intensity vs. distance with measured diameter = 290 nm]
Measurement of PSF

Measured diameter = 290 nm
Predicted diameter = 250 nm
Measurement of PSF

Measured diameter = 290 nm
Predicted diameter = 250 nm
Diameter lens alone = 1,500 nm
6.003 Approach to Increased Resolution

Uniform Illumination

Structured Illumination

Normalized intensity

Distance (μm)
Today’s Lab

Generate interesting images using the synthetic aperture approach.