Name:
Kerberos Username:

Enter all answers in the boxes provided.
Other work on pages with QR codes may be considered when assigning partial credit.

You have two hours.
This quiz is closed book, but you may use one 8.5 × 11 sheet of paper (two sides).
No calculators, computers, cell phones, music players, or other electronic devices.
1. Describing Sinusoids  [20 points]

Part 1. Let \( x_1[n] = a_1 e^{j\Omega_1 n} + a_1^* e^{-j\Omega_1 n} \) as shown in the following figure.

Estimate \( a_1 \) and \( \Omega_1 \), where \( \Omega_1 \) is real-valued and \( a_1 \) may be complex. Place an "x" on the complex plane shown below (where the circle has a radius of 1) to indicate the value of \( a_1 \). Also, place an "x" on the number line shown below to indicate the value of \( \Omega_1 \).

This signal can be written as

\[
x_1[n] = a_1 e^{j\Omega_1 n} + a_1^* e^{-j\Omega_1 n} = a_1 e^{j\Omega_1 n} + (a_1 e^{j\Omega_1 n})^* = 2 \text{Re}(a_1 e^{j\Omega_1 n}) = \text{Re}(2a_1 e^{j\Omega_1 n}).
\]

Thus this signal is the real part of a vector \( 2a_1 \) in the complex plane whose angle increases by \( \Omega_1 \) radians per sample \( n \). Since the period of \( x_1[n] \) is 20,

\[
e^{j\Omega_1 n} = e^{j\Omega_1 (n+20)} = e^{j\Omega_1 n} e^{j20\Omega_1}
\]

and the period of the complex exponential is \( 2\pi \), it follows that \( 20\Omega_1 = 2\pi \), and

\[
\Omega_1 = \frac{2\pi}{20} \approx 0.314.
\]

The peak amplitude is approximately 1, so \( |2a_1| = 1 \), and \( |a_1| = 0.5 \). The first peak of \( x_1[n] \) occurs about 2.5 samples after \( n = 0 \), which corresponds to approximately \( 2.5/20 = 1/8 \) of a cycle. So the angle of \( a_1 \) must start at approximately \( -\pi/4 \) and therefore \( \angle a_1 \approx -\pi/4 \). Thus

\[
a_1 \approx \frac{1}{2} e^{-j\pi/4}.
\]
Part 2. Let \( x_2[n] = c_2 \cos(\Omega_2 n) + d_2 \sin(\Omega_2 n) \) as shown in the following figure.

Estimate the real-valued constants \( c_2 \), \( d_2 \), and \( \Omega_2 \). Place an "x" on each of the number lines shown below to indicate these values.

We can express \( x_2[n] = c_2 \cos(\Omega_2 n) + d_2 \sin(\Omega_2 n) \) as

\[
x_2[n] = c_2 \left( \frac{e^{j\Omega_2 n} + e^{-j\Omega_2 n}}{2} \right) + d_2 \left( \frac{e^{j\Omega_2 n} - e^{-j\Omega_2 n}}{2j} \right)
\]

\[
= \frac{1}{2}(c_2 - jd_2)e^{j\Omega_2 n} + \frac{1}{2}(c_2 + jd_2)e^{-j\Omega_2 n}
\]

\[
= \text{Re}((c_2 - jd_2)e^{j\Omega_2 n})
\]

Since the period of \( x_2[n] \) is 10,

\[
e^{j\Omega_2 n} = e^{j\Omega_2 (n+10)} = e^{j\Omega_2 n} e^{j10\Omega_2}
\]

and the period of the complex exponential is \( 2\pi \), it follows that \( 10\Omega_2 = 2\pi \), and

\[
\Omega_2 = \frac{2\pi}{10} \approx 0.628.
\]

The peak amplitude of \( x_2[n] \) is approximately 1, so \( |c_2 - jd_2| = \sqrt{c_2^2 + d_2^2} = 1 \). The first peak of \( x_2[n] \) occurs at about 1/8 of a cycle. So the angle of \( c_2 - jd_2 \) must be approximately \(-\pi/4\). Therefore

\[
c_2 \approx d_2 \approx \frac{1}{\sqrt{2}} \approx 0.7.
\]
Part 3. Let $x_3[n] = \cos(\Omega_3 n + \phi_3)$ as shown in the following figure.

The period of $x_3[n]$ is approximately 30, so
$$\Omega_3 \approx \frac{2\pi}{30} \approx 0.2.$$  

The signal peaks at $n = 11$, so
$$x_3[n_0] = \cos(\Omega_3 n_0 + \phi_3) \approx 1$$
so $\phi_3 \approx -11\Omega_3 = 22\pi/30 \approx -2.3.$
**Part 4.** Let \( x_4[n] = \text{Re}(a_4 e^{j\Omega_4 n}) \) as shown in the following figure.

Estimate \( a_4 \) and \( \Omega_4 \), where \( \Omega_4 \) is real-valued and \( a_4 \) may be complex. Place an "x" on the complex plane shown below (where the circle has a radius of 1) to indicate the value of \( a_4 \). Also, place an "x" on the number line shown below to indicate the value of \( \Omega_4 \).

The period of \( x_4[n] \) is 20, so

\[
\Omega_4 = \frac{2\pi}{20} \approx 0.314.
\]

The peak amplitude is approximately 1, so \( |a_4| \approx 1 \). The first peak of \( x_4[n] \) occurs near \( n = 12.5 \), which corresponds to approximately \( 12.5/20 = 5/8 \) of a cycle, so that the angle of \( a_4 \) must be approximately \( \angle a_4 \approx -\frac{5}{8}\pi \).
2. Harmonic Aliasing [15 points]

Consider three periodic signals:

- \( x_1(t) \) with period \( T_1 = \frac{1}{11} \) seconds
- \( x_2(t) \) with period \( T_2 = \frac{1}{12} \) seconds
- \( x_3(t) \) with period \( T_3 = \frac{1}{13} \) seconds

Each of these signals contains a fundamental component (at frequency \( \omega \) given by \( \frac{2\pi}{T} \)) as well as harmonics 2, 3, 4, and 5, but not other frequencies.

Each of these continuous-time signals is sampled 40 times per second to generate corresponding discrete-time signals:

- \( x_1[n] = x_1(n/40) \)
- \( x_2[n] = x_2(n/40) \)
- \( x_3[n] = x_3(n/40) \)

Each of these discrete-time signals contains exactly five discrete-time sinusoidal components with frequencies in the range \( 0 \leq \Omega \leq \pi \).

Each plot on the facing page shows the frequencies found in one of these DT signals. In the circle next to each plot, write the name of the corresponding signal (either \( x_1 \), \( x_2 \), or \( x_3 \)).

Each of the DT frequency components is associated with one of the harmonics in the original CT signal. For each DT frequency, write the number of the associated CT harmonic (1-5) in the box above that frequency. If none of these harmonics could have produced a given frequency, enter an \( X \) in its box instead.

The fundamental component of the \( i \)th CT signal \( x_i(t) \) has the form \( e^{j2\pi t/T_i} \). Sampling this signal 40 times per second results in a DT signal of the form \( e^{j2\pi n/(40T_i)} \), which has a DT frequency of \( 2\pi/(40T_i) = \pi/(20T_i) \). Thus the frequency of the fundamental component of \( x_1[n] \) is \( \frac{11\pi}{20} \).

The second harmonic frequency is \( \frac{22\pi}{20} \), which is greater than \( \pi \). But its alias at \( 2\pi - \frac{22\pi}{20} = \frac{18\pi}{20} \) is in the range \([0, \pi]\). Thus the second harmonic of \( x_1(t) \) generates a DT frequency of \( \frac{18\pi}{20} \). The third harmonic frequency is \( \frac{33\pi}{20} \), which is greater than \( \pi \). It aliases to \( 2\pi - \frac{33\pi}{20} = \frac{7\pi}{20} \) in the range \([0, \pi]\). The fourth harmonic frequency is \( \frac{44\pi}{20} \), which is greater than \( 2\pi \). It aliases to \( \frac{44\pi}{20} - 2\pi = \frac{4\pi}{20} \) in the range \([0, \pi]\). The fifth harmonic frequency is \( \frac{66\pi}{20} \), which is greater than \( 2\pi \). It aliases to \( \frac{55\pi}{20} - 2\pi = \frac{15\pi}{20} \) in the range \([0, \pi]\).

These results for \( x_1[n] \) and analogous results for \( x_2[n] \) and \( x_3[n] \) are summarized in the table on the next page.
<table>
<thead>
<tr>
<th>signal</th>
<th>harmonic number</th>
<th>harmonic frequency</th>
<th>base-band alias $[0, \pi]$</th>
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<td>$11\pi/20$</td>
</tr>
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<td>$x_1$</td>
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<td>$33\pi/20$</td>
<td>$2\pi - 33\pi/20 = 7\pi/20$</td>
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<td>$55\pi/20 - 2\pi = 15\pi/20$</td>
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<tr>
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<tr>
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<tr>
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<td>$x_3$</td>
<td>5</td>
<td>$65\pi/20$</td>
<td>$4\pi - 65\pi/20 = 15\pi/20$</td>
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</tbody>
</table>
3. DTFT  

**[18 points]**

**Part 1.**

Consider a signal $x_1$ given by the following expression:

$$x_1[n] = \begin{cases} 1 & \text{if } |n| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Sketch the magnitude and angle of $X_1$ on the axes below. Label key points.

Starting with the definition of the Fourier Transform,

$$X_1(\Omega) = \sum_{n=-\infty}^{\infty} x_1[n]e^{-j\Omega n} = \sum_{n=-1}^{1} e^{-j\Omega n} = e^{j\Omega} + 1 + e^{-j\Omega} = 1 + 2\cos \Omega$$

This function is plotted below.

Since $X_1(\Omega)$ is real-valued, the magnitude of $X_1(\Omega)$ is equal to the absolute value of $X_1(\Omega)$ as shown in the top left plot.

Because $X_1(\Omega) < 0$ for $\frac{2\pi}{3} < |\Omega| + 2\pi i < \frac{4\pi}{3}$ (where $i$ is an integer) the angle of $X_1(\Omega)$ is $\pi$, as shown in the top right plot.
Part 2.
Consider a signal $x_2$ described by the following diagram:

$$X_2(\Omega) = \frac{6 + 2e^{-j\Omega} - 2e^{-j2\Omega} - e^{-j3\Omega}}{(3 - e^{-j2\Omega})(2 - e^{-j2\Omega})}$$

$$X_2(\Omega) = \sum_{n=-\infty}^{\infty} x_2[n]e^{-j\Omega n}$$

$$= 1e^{j\Omega 0} + \frac{1}{3}e^{-j\Omega 1} + \left(\frac{1}{3}\right)^2 e^{-j\Omega 3} + \left(\frac{1}{3}\right)^3 e^{-j\Omega 5} + \left(\frac{1}{3}\right)^4 e^{-j\Omega 7} + \ldots$$

$$+ \frac{1}{2}e^{-j\Omega 2} + \left(\frac{1}{2}\right)^2 e^{-j\Omega 4} + \left(\frac{1}{2}\right)^3 e^{-j\Omega 6} + \left(\frac{1}{2}\right)^4 e^{-j\Omega 8} + \ldots$$

$$= 1 + \frac{1}{3}e^{-j\Omega} + \frac{1}{2}e^{-j2\Omega}$$

$$= \frac{6 + 2e^{-j\Omega} - 2e^{-j2\Omega} - e^{-j3\Omega}}{(3 - e^{-j2\Omega})(2 - e^{-j2\Omega})}$$
4. More Than Meets The Eye [20 points]

Part 1. Consider the function $x_1$ described by the following expression and plot:

$$x_1(t) = 3te^{-2|t|}$$

Determine the Fourier transform $X_1$ of this signal. Express your answer in closed form.

$$X_1(\omega) = -\frac{24j\omega}{(4 + \omega^2)^2}$$

\begin{align*}
y_1(t) &= e^{-t}u(t) \quad \text{FT} \quad Y_1(\omega) = \int_0^{\infty} e^{-t}e^{-j\omega t}dt = \frac{1}{1 + j\omega} \\
y_2(t) &= e^{-2t}u(t) = y_1(2t) \quad \text{FT} \quad Y_2(\omega) = \frac{1}{2}Y_1\left(\frac{\omega}{2}\right) = \frac{1}{2 + j\omega} \\
y_3(t) &= te^{-2t}u(t) = ty_2(t) \quad \text{FT} \quad Y_3(\omega) = j\frac{d}{d\omega}Y_2(\omega) = \frac{1}{(2 + j\omega)^2} \\
y_4(t) &= te^{-2|t|} = te^{-2t}u(t) + te^{2t}u(-t) = y_3(t) - y_3(-t) \quad \text{FT} \quad Y_4(\omega) = \frac{1}{(2 + j\omega)^2} - \frac{1}{(2 - j\omega)^2} \\
x_1(t) &= 3te^{-2|t|} = 3y_4(t) \quad \text{FT} \quad X_1(\omega) = \frac{3}{(2 + j\omega)^2} - \frac{3}{(2 - j\omega)^2} = -\frac{24j\omega}{(4 + \omega^2)^2}
\end{align*}
Part 2. Now consider a signal $x_2$ whose Fourier transform $X_2$ is given by the following expression:

$$X_2(\omega) = 3\omega e^{-2|\omega|}$$

Determine a closed-form expression for $x_2(t)$.

$$x_2(t) = \frac{j12t}{\pi(4 + t^2)^2}$$

Duality: If $x(t) \mathcal{F} X(\omega)$ then

$$X(t) \mathcal{F} 2\pi x(-\omega)$$

From part 1,

$$3te^{-2|t|} \mathcal{F} -\frac{24j\omega}{(4 + \omega^2)^2}$$

By duality,

$$-\frac{24jt}{(4 + t^2)^2} \mathcal{F} 2\pi(-3\omega e^{-2|\omega|})$$

$$\frac{j12t}{\pi(4 + t^2)^2} \mathcal{F} 3\omega e^{-2|\omega|}$$
Part 3

Assume that a function $x_3$ has a Fourier transform given by $X_3$.
Let $y_3$ be defined in terms of $x_3$, as follows:

$$y_3(t) = \dot{x_3}(3(t + 5))$$

where $\dot{x_3}(t)$ is the time derivative of $x_3(t)$.

Find $Y_3(\omega)$ in terms of $X_3$:

$$Y_3(\omega) = \frac{1}{3} j \omega e^{j 5 \omega} X_3 \left( \frac{\omega}{3} \right)$$

Let

$$z(t) = \dot{x}(t) = \frac{dx(t)}{dt}.$$  

Then

$$Z(\omega) = j \omega X(\omega).$$

We can express $y(t)$ in terms of $z$ as

$$y(t) = z(3(t + 5))$$

and then

$$Y(\omega) = \int y(t)e^{-j \omega t}dt$$

$$= \int z(3(t + 5))e^{-j \omega t}dt$$

Let $\tau = 3(t + 5)$ then $d\tau = 3dt$ and

$$Y(\omega) = \int z(\tau)e^{-j \omega (\frac{\tau}{3} - 5)} \frac{1}{3} d\tau$$

$$= \frac{1}{3} e^{j 5 \omega} \int z(\tau)e^{-j \frac{\omega}{3} \tau} d\tau$$

$$= \frac{1}{3} e^{j 5 \omega} Z \left( \frac{\omega}{3} \right)$$

$$= \frac{1}{3} e^{j 5 \omega} j \frac{\omega}{3} X \left( \frac{\omega}{3} \right)$$
5. Dome, Sweet Dome [27 points]

Ben Bitdiddle created a signal $x_0[n]$ representing the MIT dome, but he only saved the DTFS coefficients $X_0[k]$ (and not the original signal). However, he knew that one period of the original signal (which is periodic in $N = 51$) looked like this:

Ben tried several different methods of recovering the original image based on $X_0[k]$, by applying the DTFS synthesis equation to the following sets of coefficients.

For each set of Fourier coefficients described below ($X_A$ through $X_I$), determine the corresponding signal on the following page ($x_1$ through $x_{24}$). Assume that all of the signals on the following page are purely real and are periodic in $N = 51$. If the required signal would be complex-valued, write COMPLEX in the box; otherwise, write the name of the signal from the following page.

The original signal is periodic in $N = 51$ as shown below.
Part 1. \(X_A[k] = \text{Re}(X_0[k])\)

\[x_A = \begin{array}{c}
\end{array}\]

\[X_A[k] = \text{Re}(X_0[k]) = \frac{1}{2}X_0[k] + \frac{1}{2}X_0^*[k]\] (property of complex numbers)

Now find the effect of conjugating \(X[k]\).

\[X[k] = \frac{1}{N} \sum x[n]e^{-j\frac{2\pi kn}{N}}\] (Fourier analysis equation)

\[X^*[k] = \frac{1}{N} \sum x^*[n]e^{j\frac{2\pi kn}{N}}\] (conjugate both sides)

\[X^*[k] = \frac{1}{N} \sum x^*[-n]e^{-j\frac{2\pi kn}{N}}\] \((n \rightarrow -n)\)

\[x^*[-n] \Leftrightarrow X^*[k]\] (Fourier analysis equation)

Then

\[x_A[n] = \frac{1}{2}x_0[n] + \frac{1}{2}x_0^*[-n] = \frac{1}{2}x_0[n] + \frac{1}{2}x_0[-n]\]

since \(x_0[n]\) is real-valued. The flipped signal \(x_0[-n]\) looks a lot like \(x_0[n]\) (since that function is symmetric about \(n = 18.5\)) but it is shifted by 15 samples. Thus when \(x_0[n]\) is added to \(x_0[-n]\), part of the dome from \(x_0[n]\) overlaps part of the dome from \(x_0[-n]\). The result looks like \(x_{16}[n]\).

We can think about symmetry properties as a way to check this answer. The sum of \(x_0[n]\) and \(x_0[-n]\) (which is a flipped version about \(n = 0\)) will be an even function of \(n\). Since \(x_0[n]\) is also periodic in \(n = 51\), the result of adding \(x_0[n]\) to \(x_0[-n]\) is also symmetric about \(n = 25.5\). There are only four signals with this symmetry: \(x_9, x_{11}, x_{16}, \) and \(x_{22}\). (Notice that \(x_{14}\) is not quite right since there are only four leading values of zero.) However, the signal is clearly not zero, eliminating \(x_{11}\). Also \(x_9\) is upside down and \(x_{22}\) is upside-down plus a constant. Thus the answer must be \(x_{16}\).
Part 2. \( X_B[k] = \text{Im} \left( X_0[k] \right) \)

\[
X_B[k] = \text{Im}(X_0[k]) = \frac{1}{2j} X_0[k] - \frac{1}{2j} X_0^*[k] \quad \text{(property of complex numbers)}
\]

Then

\[
x_B[n] = \frac{1}{2j} x_0[n] - \frac{1}{2j} x_0^*[-n] = \frac{1}{2j} x_0[n] - \frac{1}{2j} x_0[-n]
\]

Since \( x_0[n] \) is real-valued, \( x_B[n] \) must be complex-valued.

None of the possible answers are complex valued, so the answer is "COMPLEX".

Part 3. \( X_C[k] = j \text{Im} \left( X_0[k] \right) \)

\[
X_C[k] = j \text{Im}(X_0[k]) = \frac{1}{2} X_0[k] - \frac{1}{2} X_0^*[k] \quad \text{(property of complex numbers)}
\]

Thus

\[
x_C[n] = \frac{1}{2} x_0[n] - \frac{1}{2} x_0^*[-n] = \frac{1}{2} x_0[n] - \frac{1}{2} x_0[-n]
\]

since \( x_0[n] \) is real-valued. When \( x_0[-n] \) is subtracted from \( x_0[n] \), the result is an odd function of \( n \). Since \( x_0[n] \) is also periodic in \( N = 51 \), the result is also antisymmetric about \( n = 25.5 \). The result looks like \( x_8[n] \).

\( x_{11} \) has the right symmetry properties, but our answer is clearly not zero. Also \( x_{13} \) clearly has the wrong shape. \( x_{21} \) is the negative of the right answer, i.e., \( x[-n] - x[n] \).

So the answer must be \( x_8 \).
Part 4. \( X_D[0] = 0, X_D[k] = X_0[k] \) otherwise

\[
x_D = \begin{cases} x_6 & \text{otherwise} \\
\end{cases}
\]

By setting \( k = 0 \) in the analysis equation,
\[
X_0[k] = \frac{1}{N} \sum x_0[n] e^{-j2\pi kn/N}
\]
we can see \( X_0[0] \) is the average value of \( x_0[n] \). Let \( \bar{x} \) represent the average value of \( x_0[n] \). Then by linearity
\[
x_0[n] - \bar{x} \iff X_0[k] - X_0[0]
\]
Setting \( X_0[0] \) to zero is thus equivalent to subtracting the average value of \( x_0[n] \) from \( x[n] \) for all \( n \).

Two signals \( x_6[n] \) and \( x_{19}[n] \) are simple vertical shifts of \( x_0[n] \). Since \( x_{19}[n] \) is shifted in the wrong direction, the answer must be \( x_6[n] \).

Part 5. \( X_E[25] = 0, X_E[k] = X_0[k] \) otherwise

\[
x_E = \text{COMPLEX}
\]

Setting the twenty-fifth component of the Fourier series to zero is equivalent to subtracting a complex exponential with frequency of \( \frac{2\pi 25}{51} \) from \( x_0[n] \).

So our new signal would be \( x_E[n] = x_0[n] - X_0[25] e^{j2\pi (25/51)n} \). Unless \( X_0[25] = 0 \), this extra term will be complex-valued.

Part 6. \( X_F[k] = X_0[k] + 1/51 \)

\[
x_F = \begin{cases} x_{20} & \text{otherwise} \\
\end{cases}
\]

By linearity, adding a constant to \( X_0[k] \) adds a signal \( y[n] \) to \( x_0[n] \) where \( y[n] \) is the signal whose Fourier series \( Y[k] \) is \( 1/51 \) for all \( k \):
\[
y[n] = \sum \frac{1}{51} e^{j\omega kn/51}
\]
By orthogonality, \( y[n] \) must be \( \delta[n] \) since the above sum goes to zero except at \( n = 0 \). Thus the solution is \( x_{20}[n] \).
Part 7. \( X_G[k] = e^{j\pi}X_0[k] \)

\[ x_G = x_{23} \]

The multiplier \( e^{j\pi} \) is equal to -1. Therefore the new signal is flipped about the horizontal axis. The solution must be \( x_{23}[n] \).

Part 8. \( X_H[0] = X_0[0], \) \( X_H[k] = e^{j\pi}X_0[k] \) otherwise

\[ x_H = x_{15} \]

The multiplier here is the same as in Part 7.

However, the DC term is still that of the original signal (which is positive). The resulting effect is that \( x_H[n] = 2X_0[0] - x_0[n] \) (i.e., it is reflected about the horizontal axis, and then shifted to account for the change in DC value).

The solution is \( x_{15}[n] \).

Part 9. \( X_I[k] = |X_0[k]|e^{j(-\angle X_0[k])} \)

\[ x_I = x_{10} \]

Negating the angle of a complex number while holding the magnitude constant has the same effect as taking the complex conjugate of the original number. This follows from thinking about the definition of magnitude and angle of a complex number \( a \):

\[ a = |a|e^{j\angle a} \]

\[ a^* = |a|e^{-j\angle a} \]

Thus \( X_I[k] = X_0^*[k] \).

Conjugating the Fourier series has the effect of conjugating the time function and then flipping it about \( n = 0 \). Since \( x_0[n] \) is real-valued, the result is just a time flip, and the answer is \( x_{10} \).
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